

Synthesizing Real-Time Schedulability Tests using Evolutionary Algorithms: A Proof of Concept

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Introduction

- Real-Time Systems
 - ▶ Characterised by the need for both functional and timing correctness
- Verifying timing correctness: Typically two stage process
 - ▶ **Timing analysis** – characterises the amount of time each task can take to execute, or message can take to be transmitted
 - ▶ **Schedulability analysis** - aims to characterise the worst-case end-to-end response time of functionality involving one or more tasks or messages, taking into account the way in which they are scheduled
- How are schedulability tests derived?

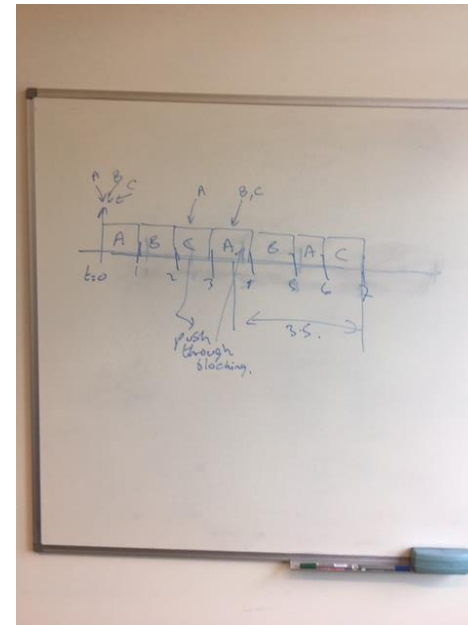
How are schedulability tests derived?



Lots of thinking?



Trying out different equations using pen and paper?



Checking simple schedules on a whiteboard

- How do you do this?

Motivation for this work

- Researchers
 - ▶ Have produced lots of great work on schedulability analysis
 - ▶ Also get things wrong
- Schedulability analysis literature many flawed proofs
 - ▶ Analysis for CAN from 1994/5 - flaws discovered and fixed in 2007
 - ▶ Analysis for self-suspending tasks from 2010 – flaws discovered in 2015 much subsequent theory also shown to be flawed in 2018
 - ▶ Analysis of wormhole routing on Network-on-Chip from 2008 – flaws discovered in 2016, revised analysis aiming to fix the problem also flawed!

- Work on formal proof can help



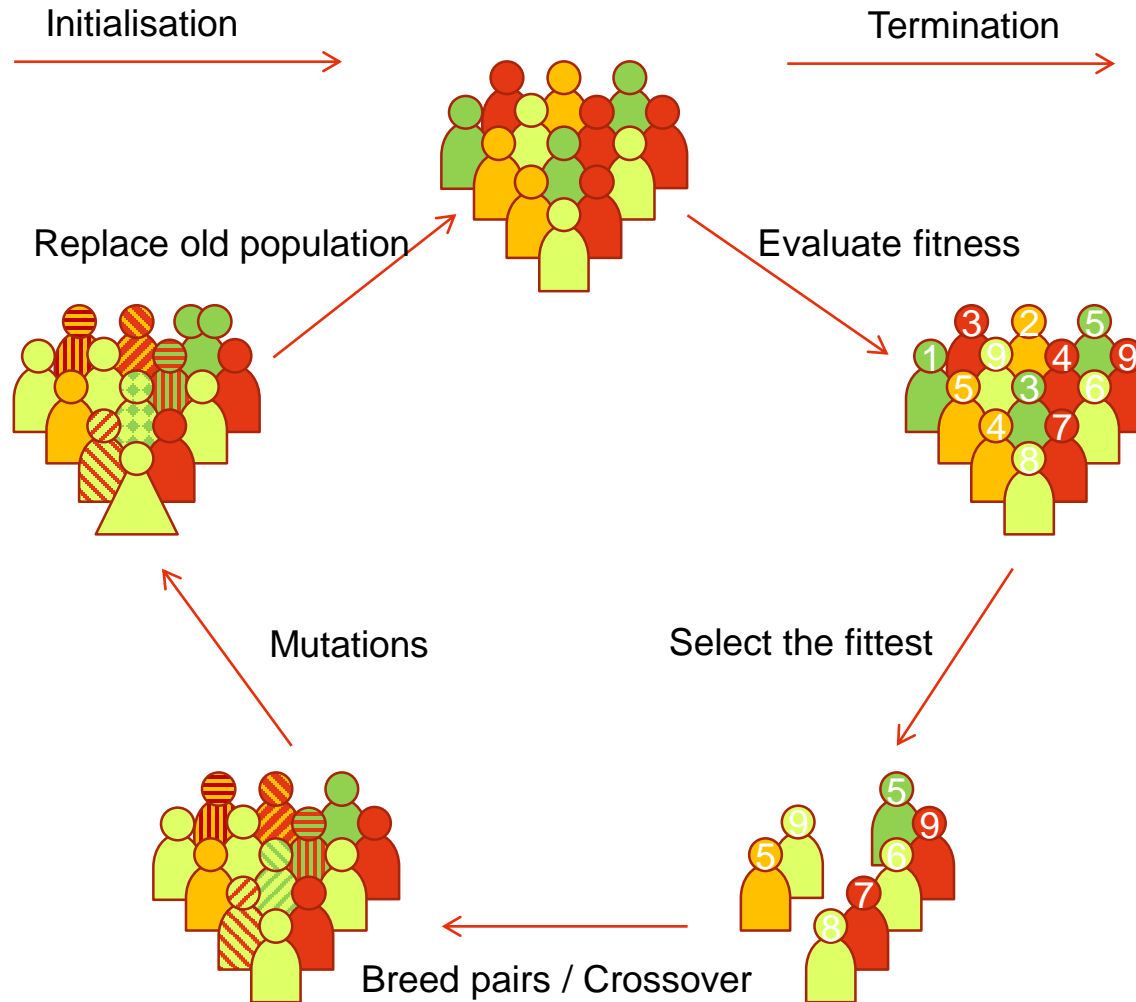
But we need to come up with analysis in the first place

The Idea

- Formulation Assistance

- ▶ Provide mechanised assistance to researchers in the formulation of schedulability tests
- ▶ Use evolutionary algorithms to semi-automate deriving response time equations

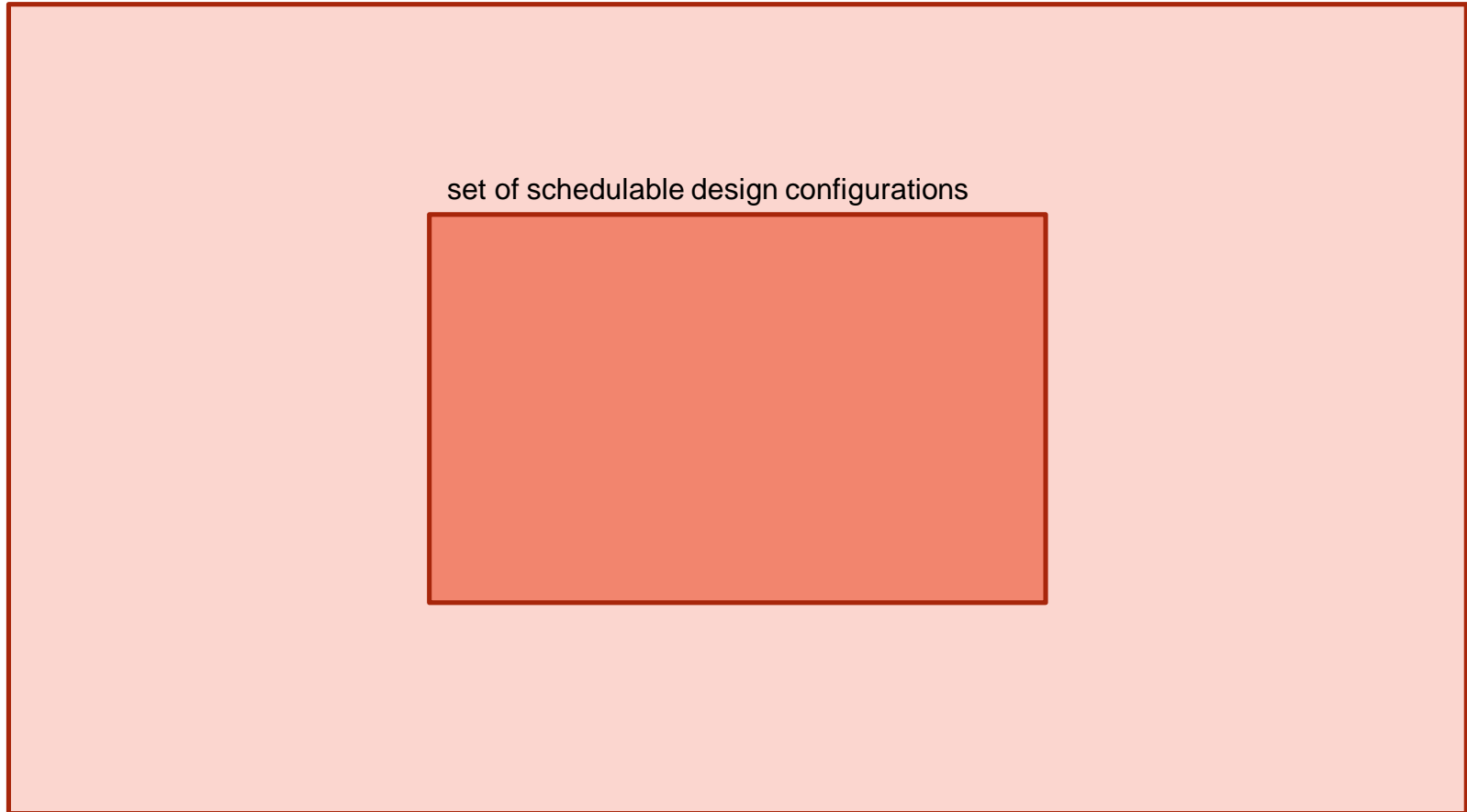
Evolutionary Algorithm



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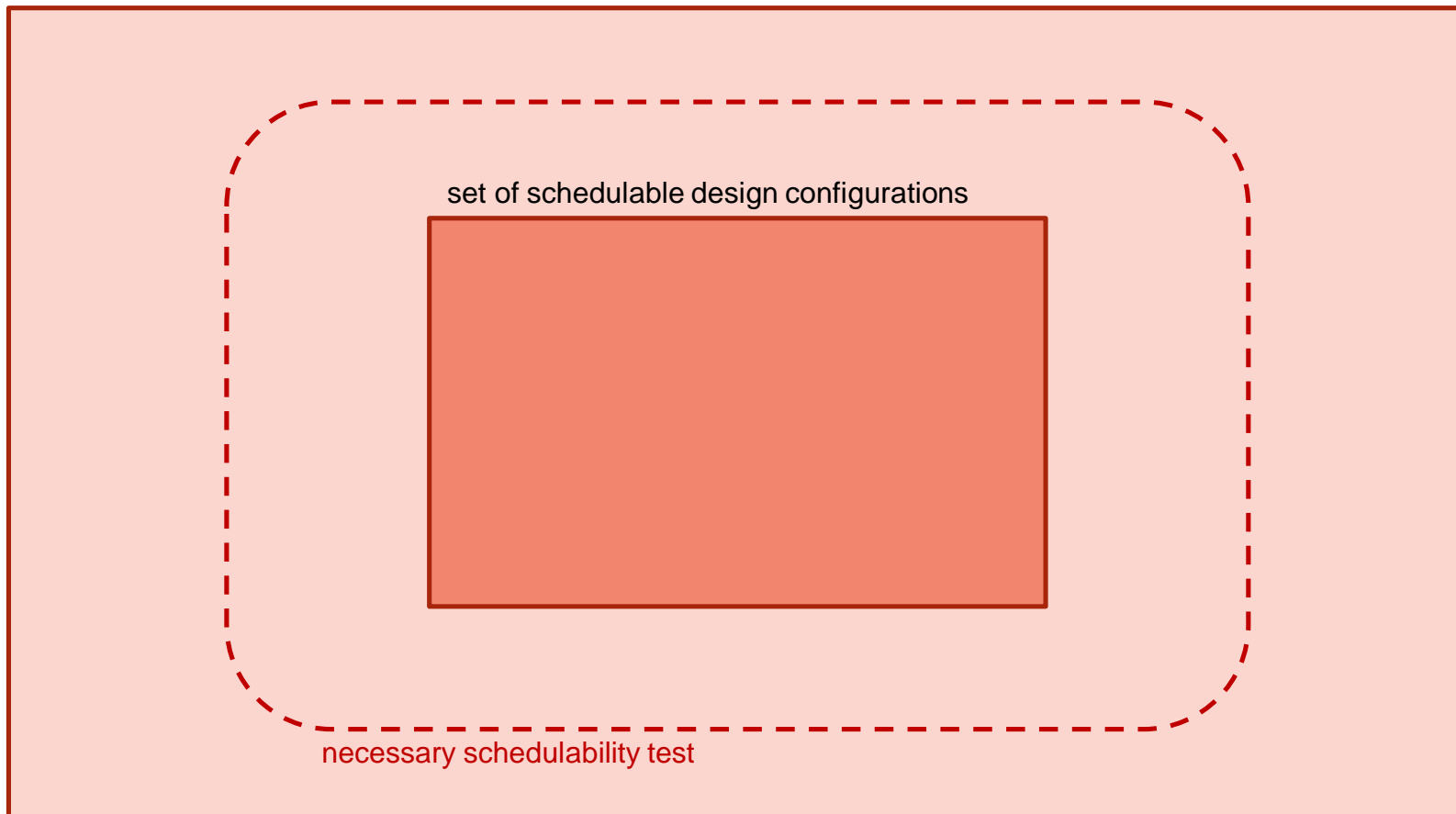


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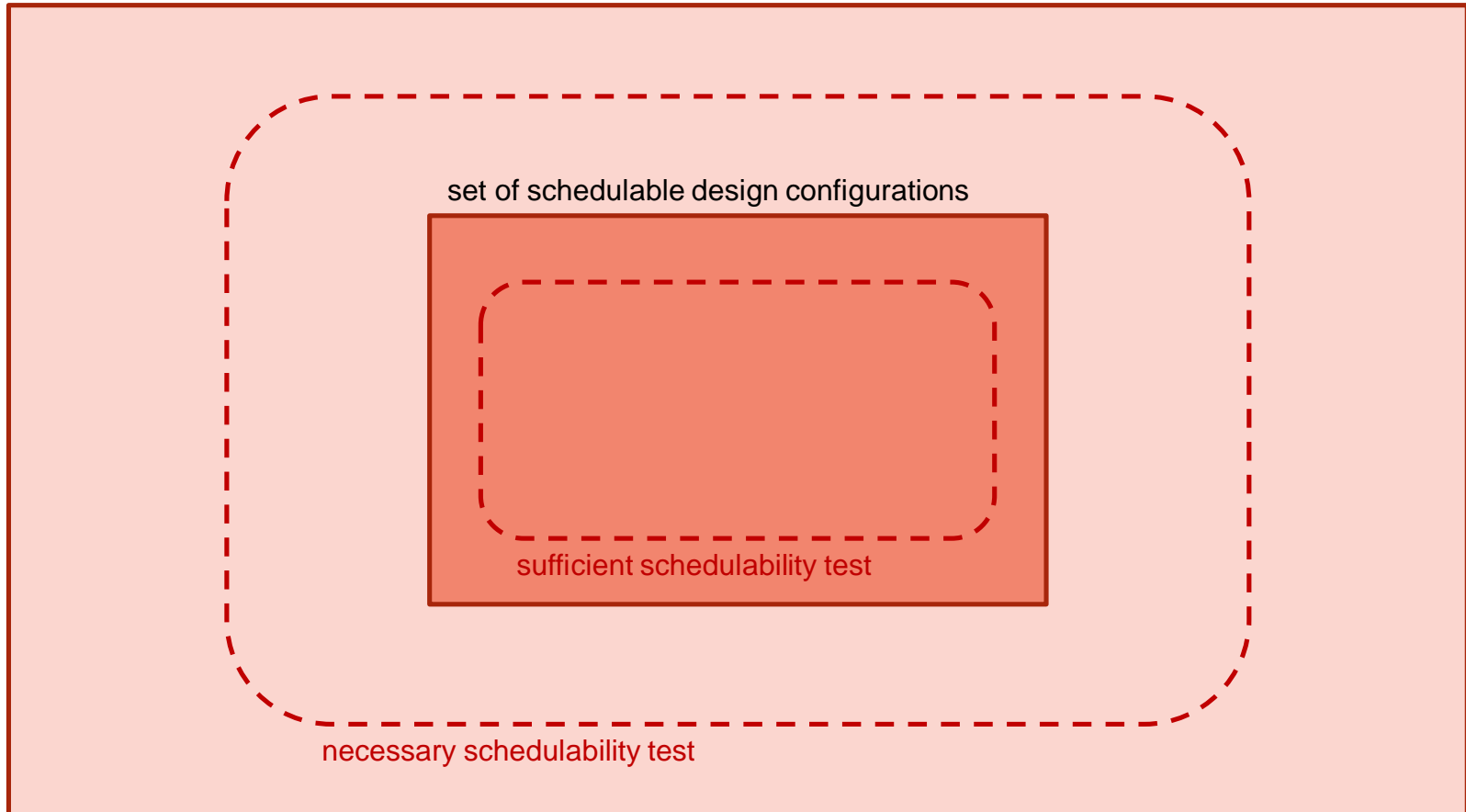


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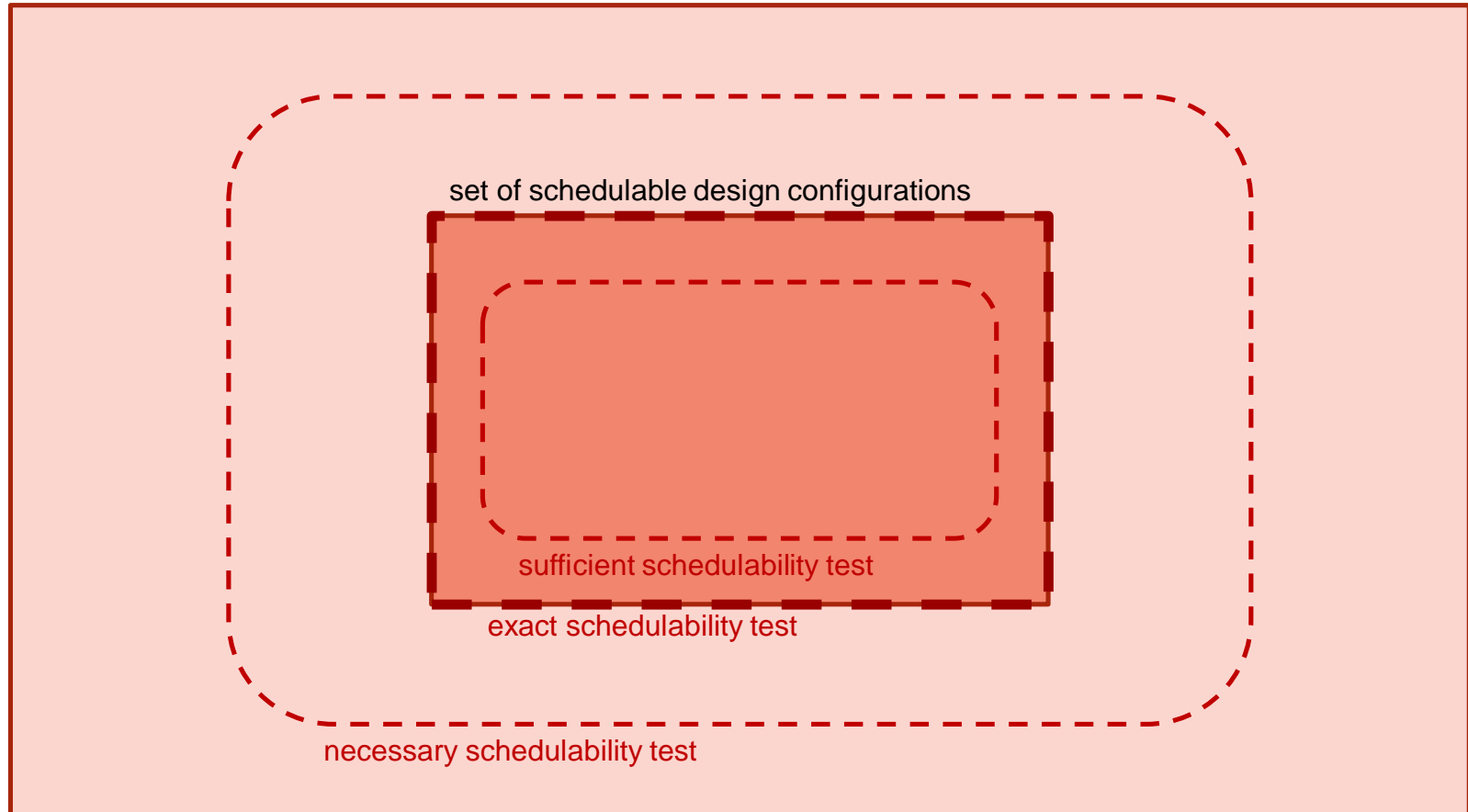
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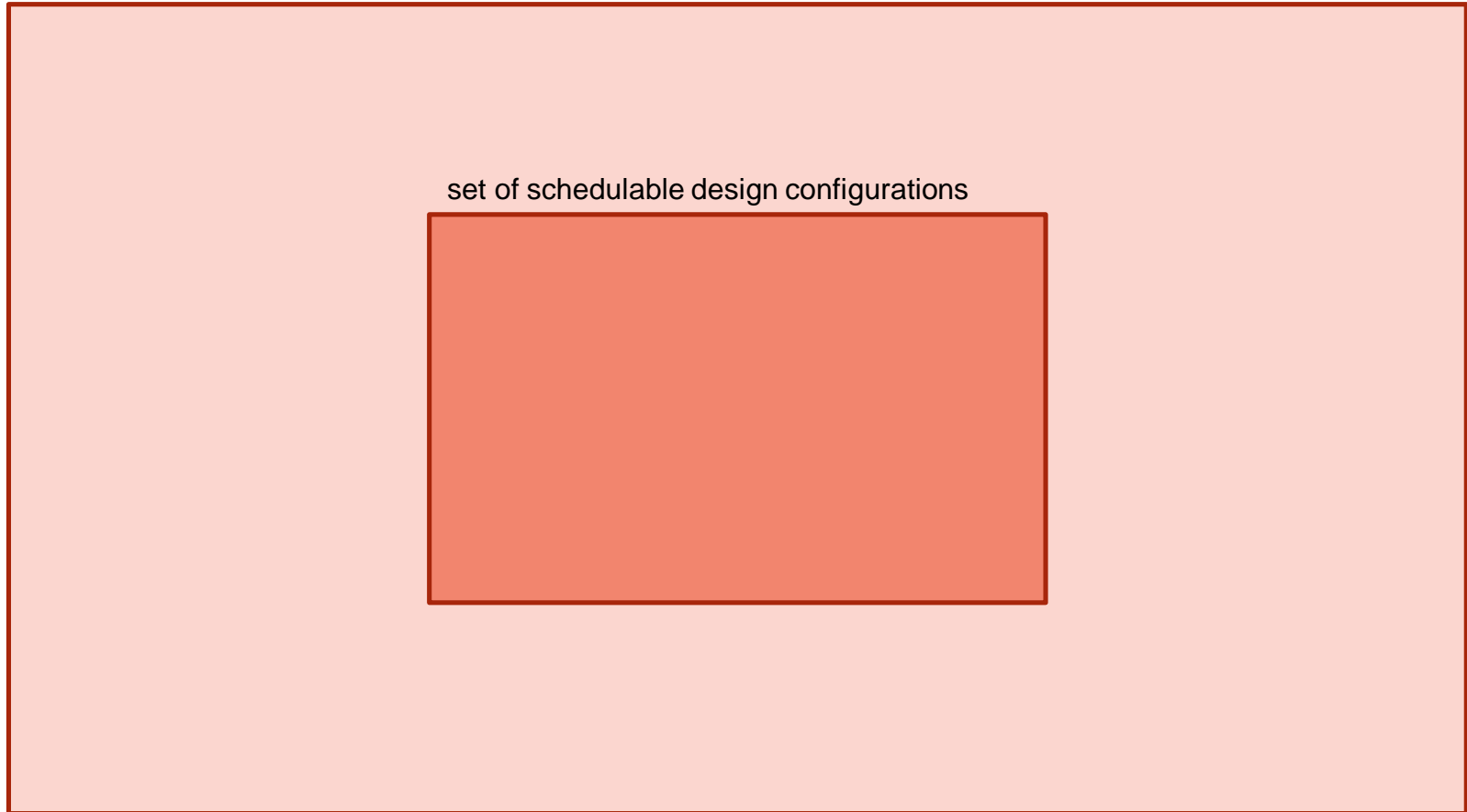
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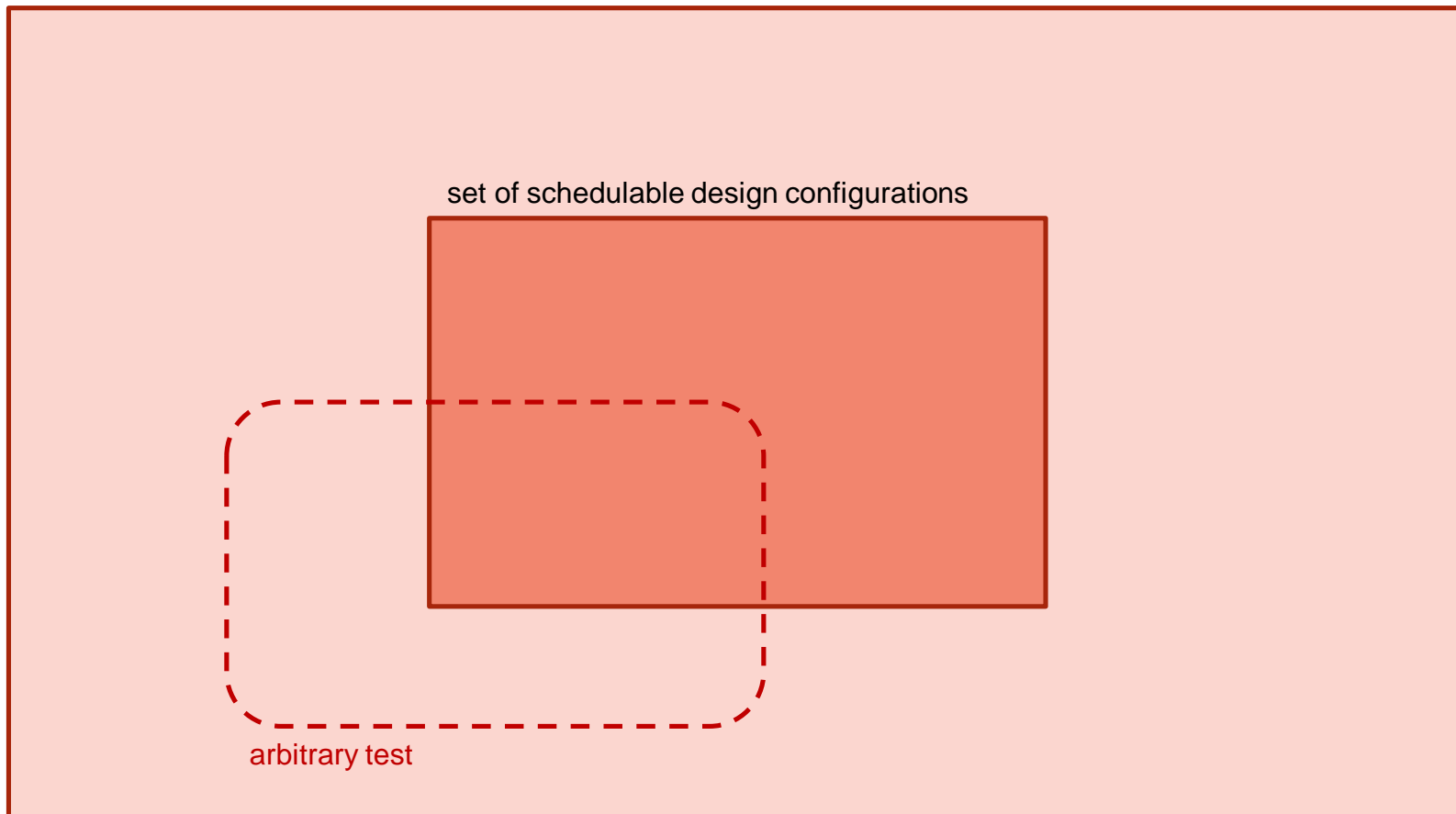


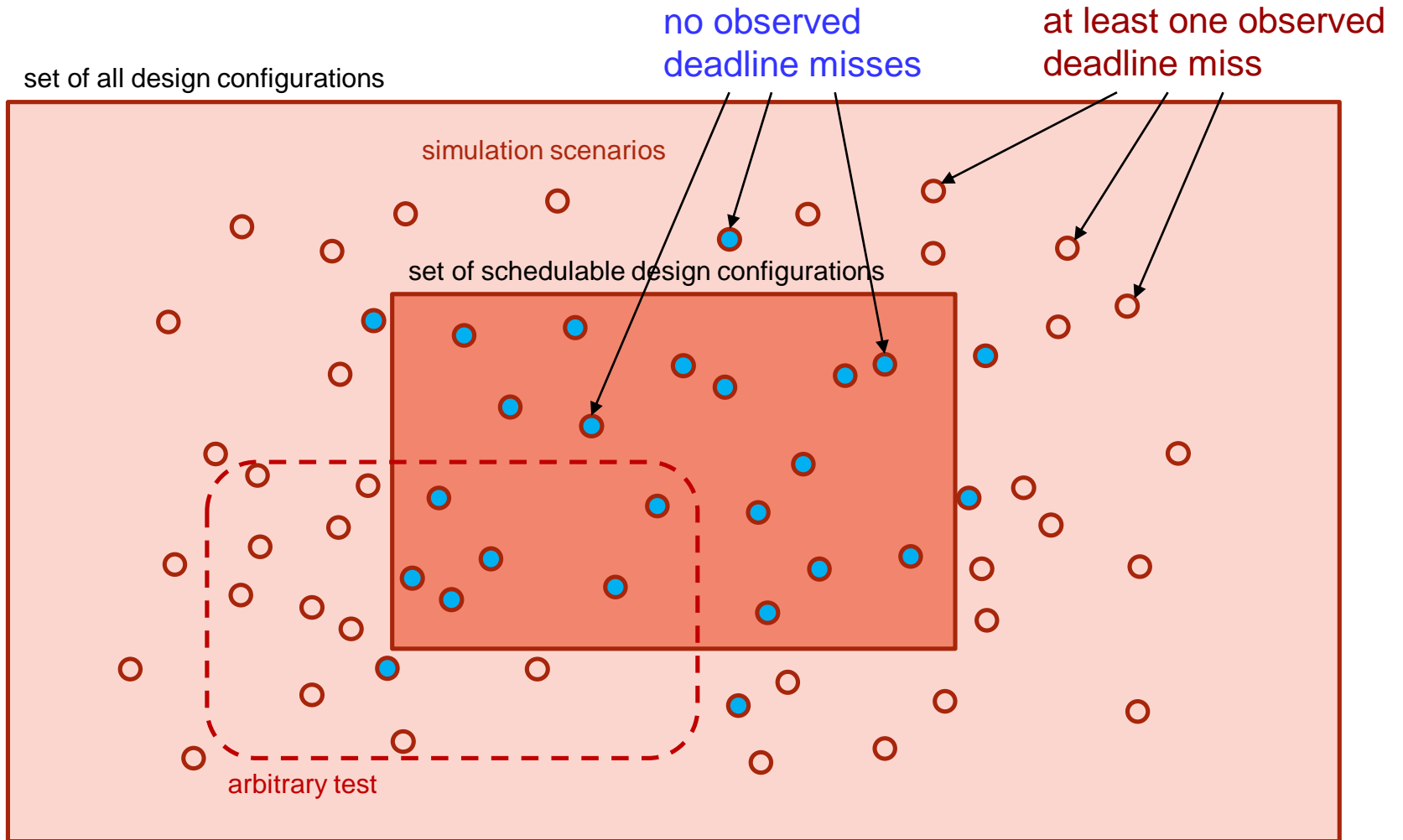
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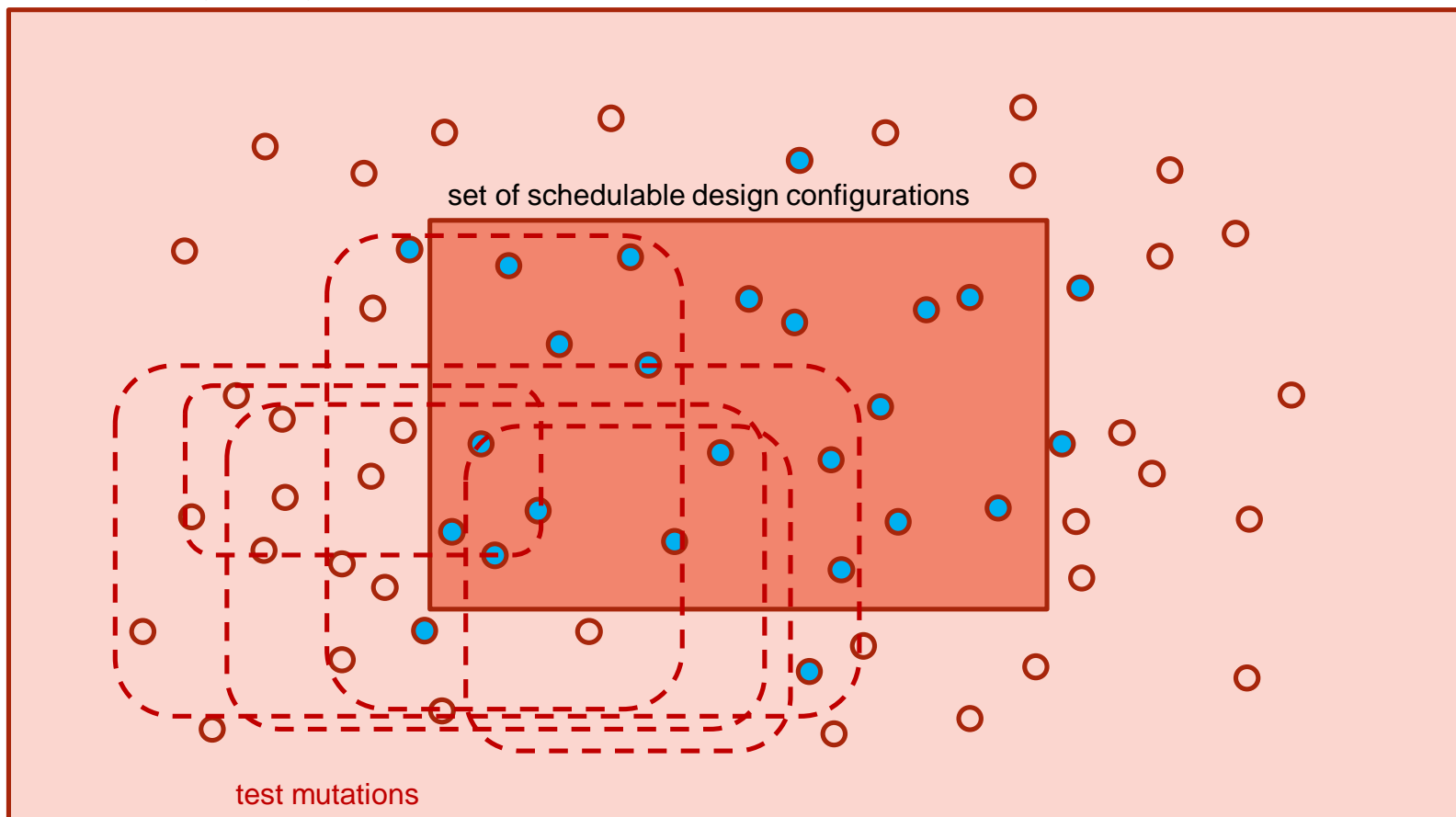
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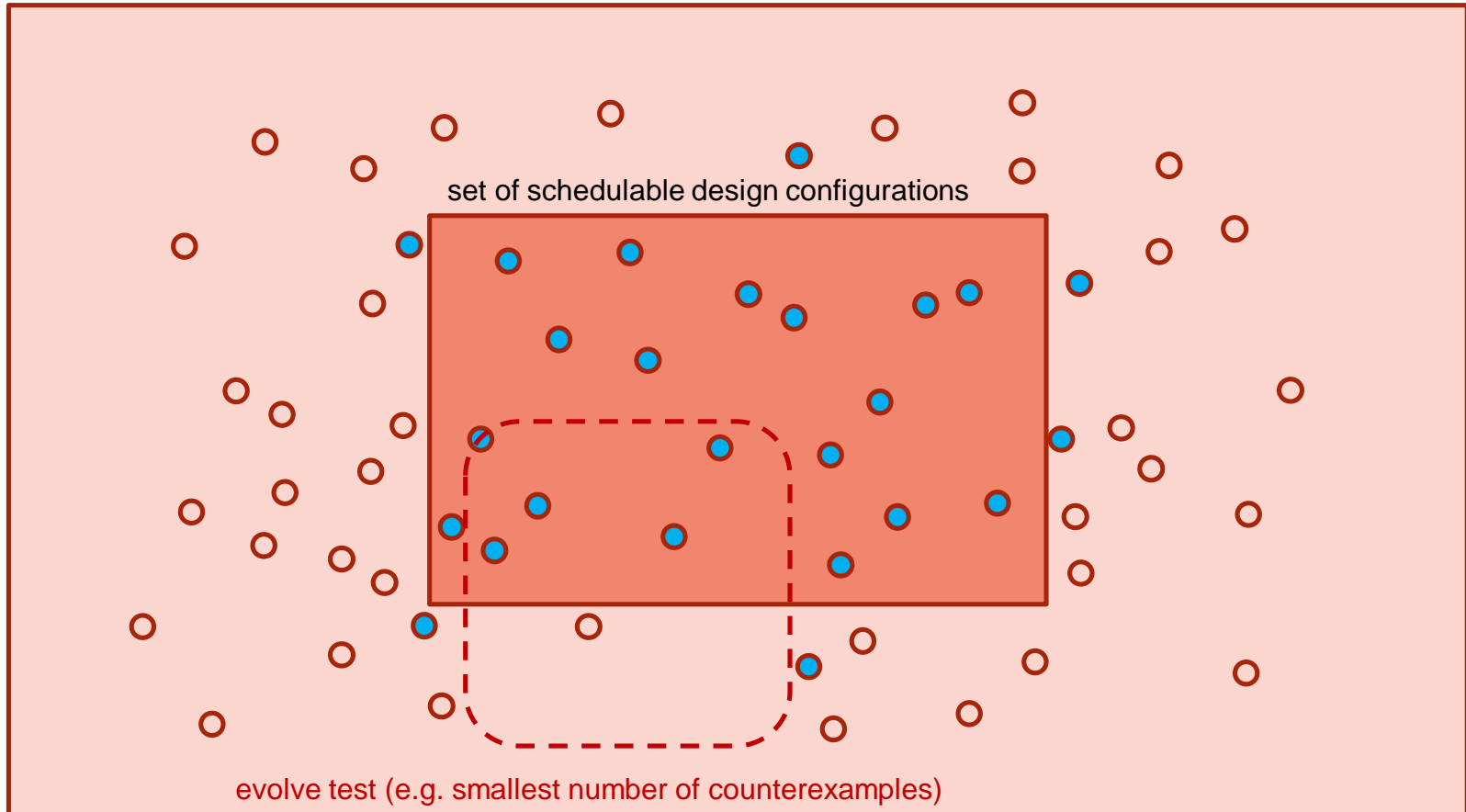




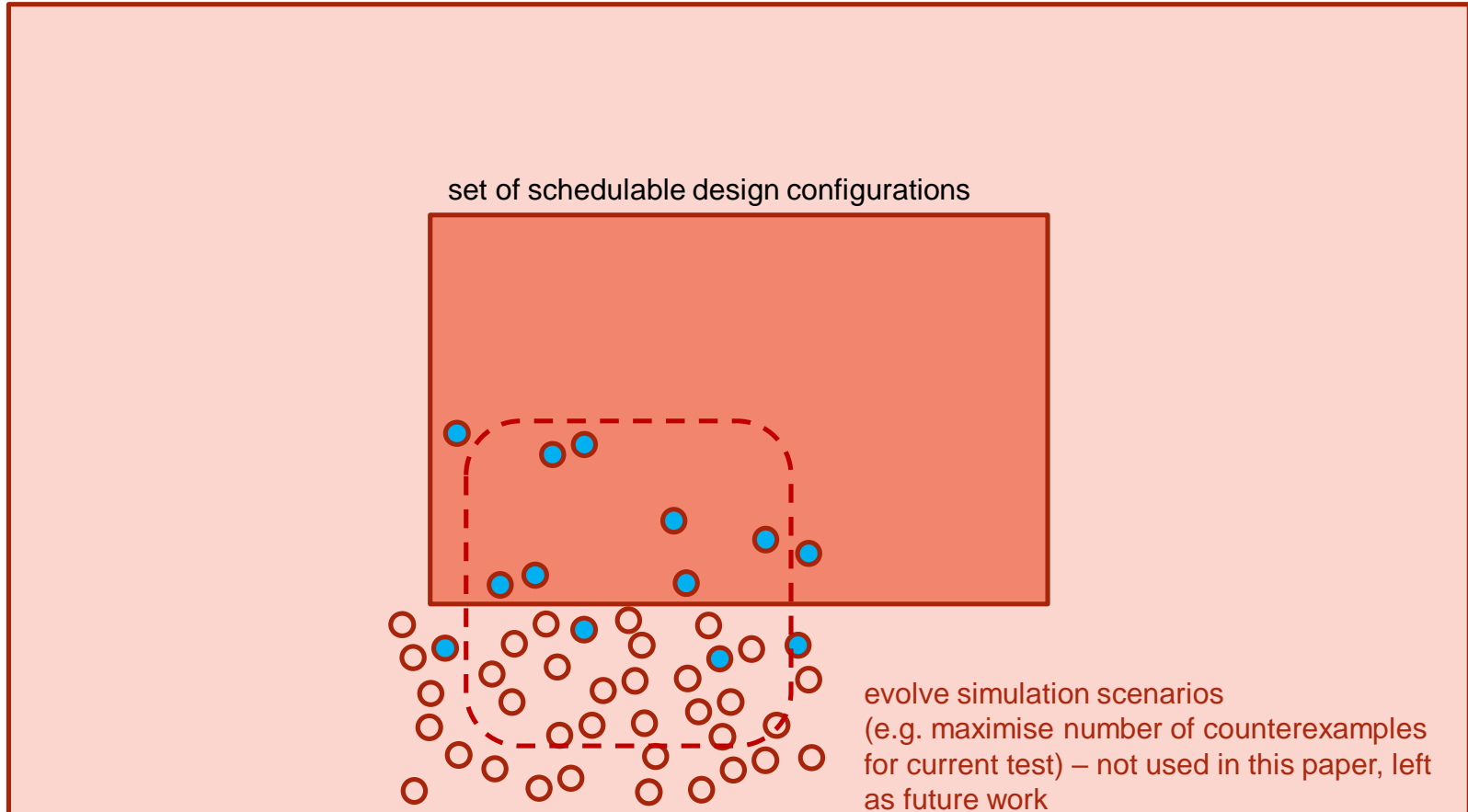
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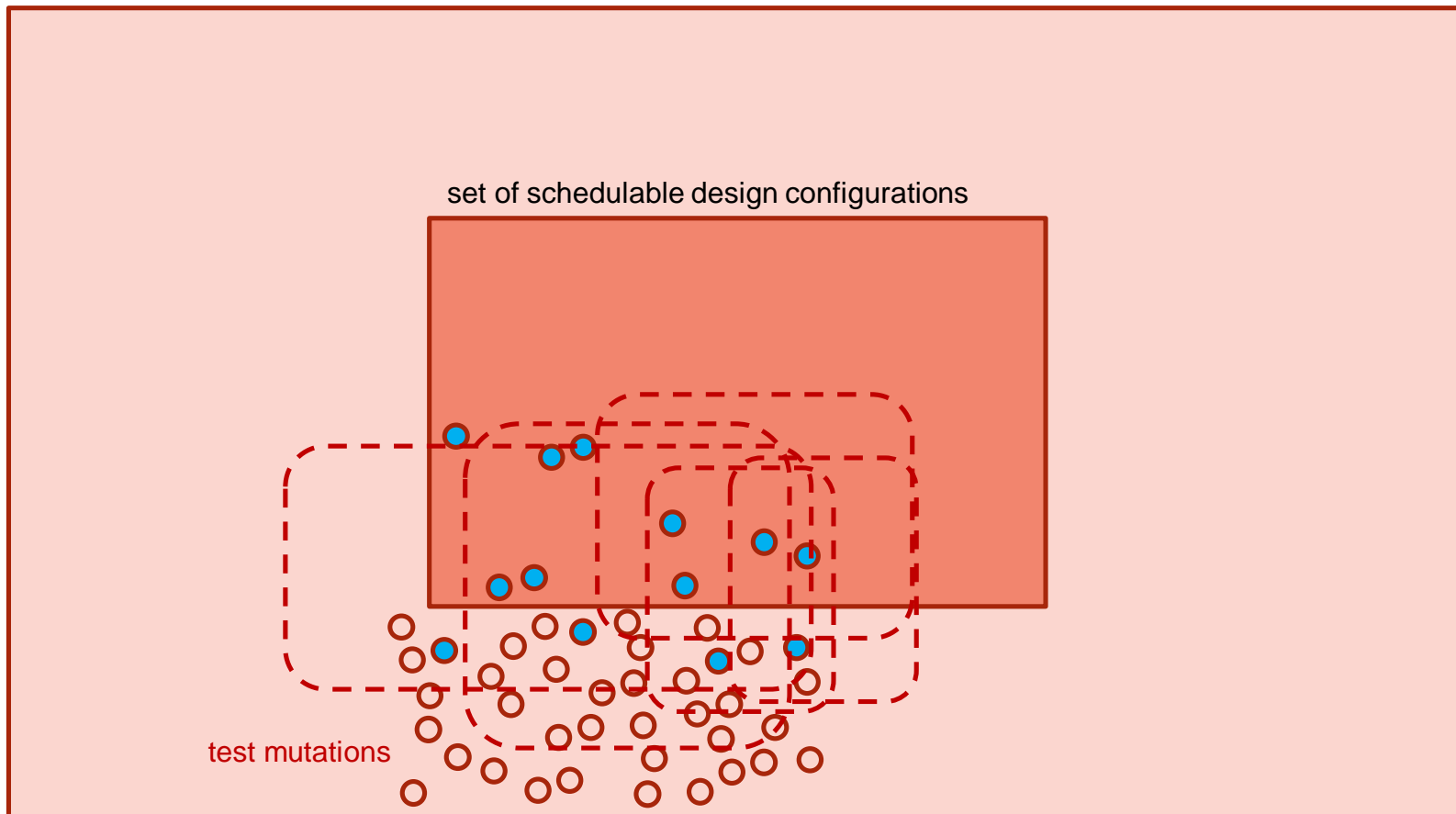
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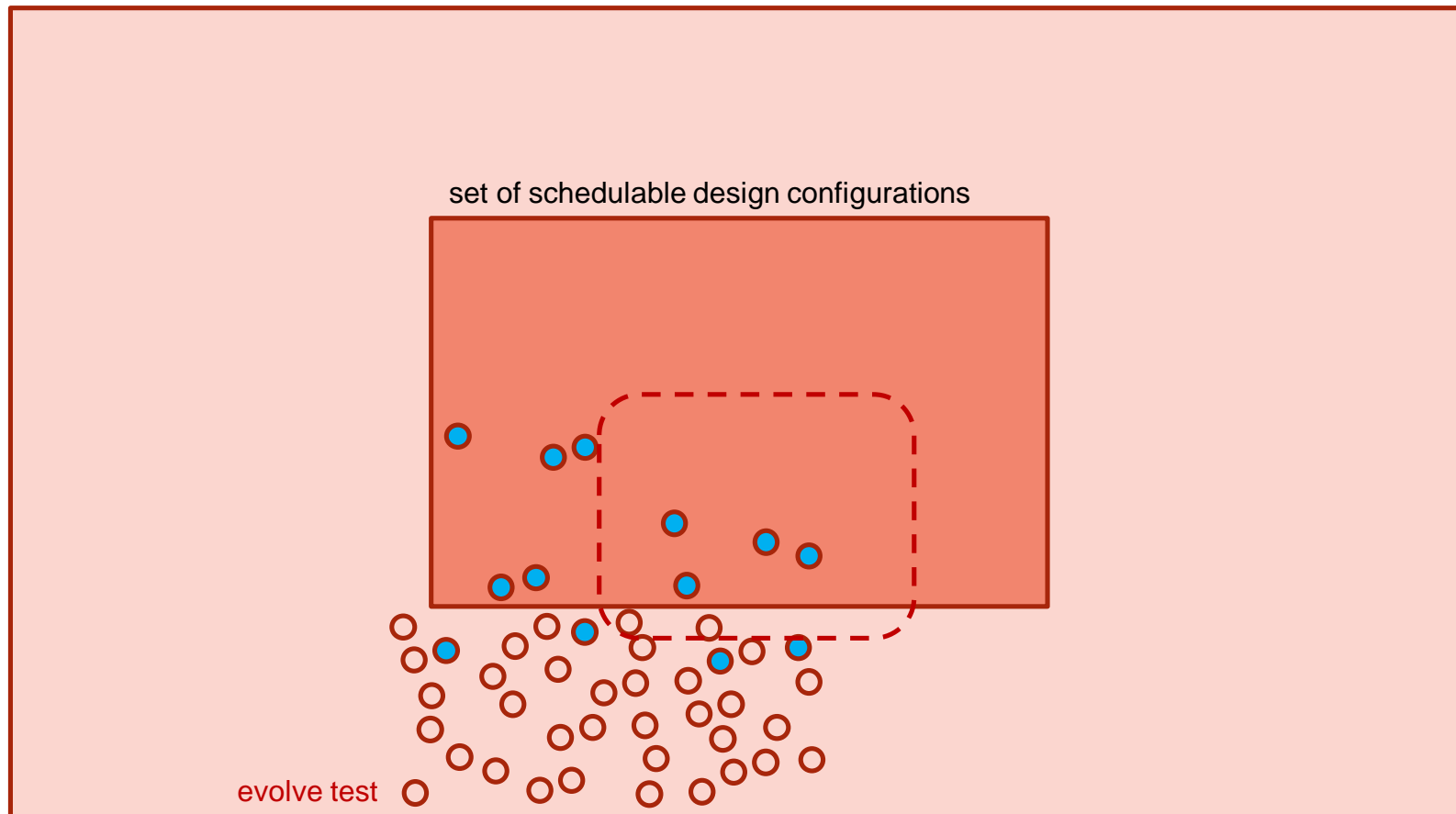
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Formulation Assistance in detail

▪ Researchers

- ▶ Consider the system model and scheduling policy
- ▶ Determine a set of *symbols* and *operators* forming a *grammar* that can be used to express candidate response time analysis equations
 - Examples of symbols: C_i , D_i , T_i , J_i , B_i , R_i
 - Examples of operators: +, -, *, /, ceiling, floor, max, min, sum
- ▶ Obtains a set of *verification vectors*
- ▶ Each verification vector represents a concrete system
 - Includes the parameters for each schedulable entity (task, message etc.) as well as their indicative response times
- ▶ Determine indicative response times
 - Measurements from a real system
 - Cycle accurate simulation
 - Simulation using a high level model

Formulation Assistance in detail

■ Grammatical Evolution

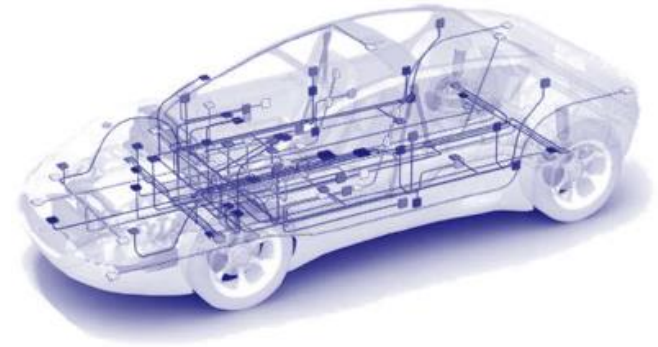
- ▶ Introduced in late 1990s by Ryan and O'Neil, similar to Genetic Programming
- ▶ Performs the evolutionary process on candidate equations that are expressions represented by variable length strings encoded according to a grammar defined in Backus-Naur Form (BNF)
- ▶ Strings are evolved via recombination and mutation that respects the specific rules of the grammar
- ▶ Advantage that the grammar provides direct control over how symbols and operators can be combined enabling domain knowledge to be applied
- ▶ Population-based approach supports parallelism
- ▶ Requires a *grammar* and *fitness function* defined by the researcher

Proof-of-Concept

▪ Controller Area Network (CAN)

- ▶ Well-known in-vehicle network used in automotive
- ▶ Famously the original analysis was flawed
- ▶ Problems identified and fixed in 2007

“Controller area network (CAN) schedulability analysis: Refuted, revisited and revised”



Proof-of-Concept

Controller Area Network (CAN)

- ▶ Exact test E1:
 - Complex two stage test (see paper)

- ▶ Sufficient Test **S1**:

$$R_i = J_i + C_i + \max(B_i, C_i) + \sum_{k \in hp(i)} \left(\left\lfloor \frac{R_i - J_i - C_i + J_k}{T_k} \right\rfloor + 1 \right) C_k$$

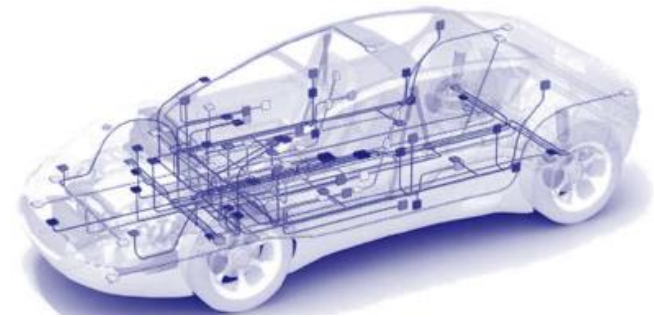
- ▶ Sufficient Test **S2**:

$$R_i = J_i + C_i + \max(B_i, C_i) + \sum_{k \in hp(i)} \left(\left\lfloor \frac{D_i - J_i - C_i + J_k}{T_k} \right\rfloor + 1 \right) C_k$$

- ▶ Flawed Test **F1**:

$$R_i = J_i + C_i + B_i + \sum_{k \in hp(i)} \left(\left\lfloor \frac{R_i - J_i - C_i + J_k}{T_k} \right\rfloor + 1 \right) C_k$$

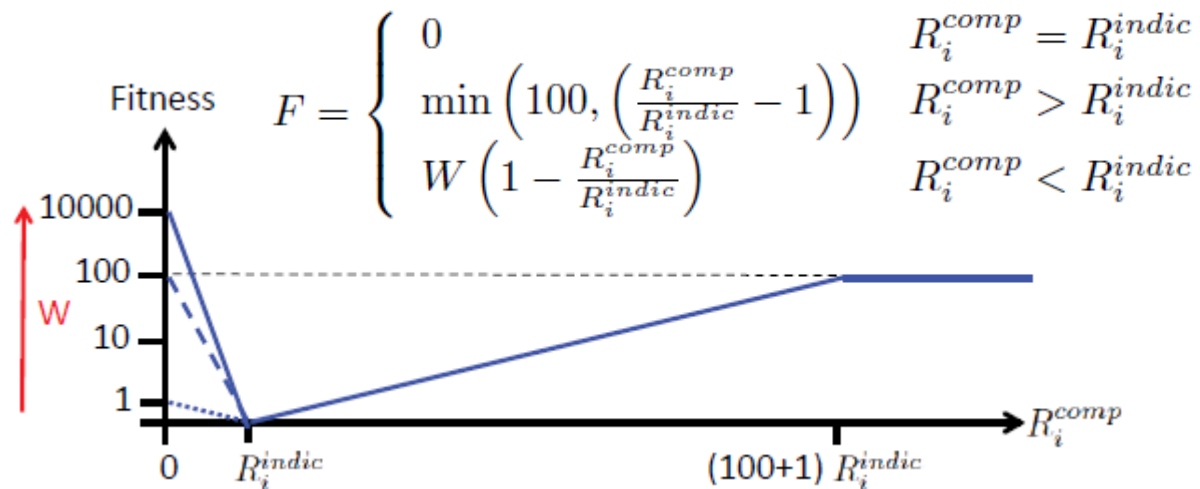
- ▶ Proof of concept aims to evolve sufficient tests of the form $R_i = \langle \text{expr} \rangle$, (can be recursive like **S1** or non-recursive like **S2**)



Proof-of-Concept: Fitness

■ Fitness function

- ▶ Compares response time computed for each message using the candidate expression with an indicative response time stored in the verification vectors
- ▶ Fitness summed for every message in each of 100 verification vectors



- ▶ Weighting factor W increases from 1 to 10,000 over generations to penalise under-approximation as evolution progresses

Proof-of-Concept: Grammar

■ Dimensionality and Scale Invariance

- ▶ Must ensure that expressions generated have the correct dimensionality (i.e. are in the same units as R_i for example $R_i = D_i$ is dimensionally correct whereas $R_i = D_i * D_i$ is not
- ▶ Also need to ensure scale invariance so that the expressions work correctly even if the measurement scale for all parameters is changed
- ▶ Avoid use of individual ceiling, floor, and multiply operators, instead use compound operators $[A/B] C$ and $[A/B] C$ that preserve dimensionality
- ▶ Reduces the size of the search space eliminating all dimensionally incorrect expressions

■ Constraints

- ▶ Embed domain knowledge in the grammar to constrain where certain symbols and operators can be used
- ▶ Response times can only be composed of multiples of C_i, C_k, J_i, B_i all other symbols can only be used in their multipliers

Proof-of-Concept: Grammar

```

<test> ::=  $R_i = \langle \text{expr} \rangle$ 
<expr> ::=
  ( $\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle$ ) |  $\langle \text{iVar} \rangle$  |
   $\langle \text{expr} \rangle * (\langle \text{exprInFloorCeil} \rangle \$ \langle \text{exprInFloorCeil} \rangle)$  |
   $\langle \text{expr} \rangle * (\langle \text{exprInFloorCeil} \rangle / \langle \text{exprInFloorCeil} \rangle)$  |
   $\text{sigma}(\langle \text{range} \rangle) (\langle \text{exprInSum} \rangle)$  |  $((\langle \text{expr} \rangle)^\sim (\langle \text{expr} \rangle))$  |
   $((\langle \text{expr} \rangle)_\sim (\langle \text{expr} \rangle))$ 
<exprInSum> ::=
  ( $\langle \text{exprInSum} \rangle \langle \text{op} \rangle \langle \text{exprInSum} \rangle$ ) |  $\langle \text{iVar} \rangle$  |  $\langle \text{kVar} \rangle$  |
   $\langle \text{exprInSum} \rangle * (\langle \text{exprInSumFloorCeil} \rangle \$ \langle \text{exprInSumFloorCeil} \rangle)$  |
   $\langle \text{exprInSum} \rangle * (\langle \text{exprInSumFloorCeil} \rangle / \langle \text{exprInSumFloorCeil} \rangle)$  |
   $((\langle \text{exprInSum} \rangle)^\sim (\langle \text{exprInSum} \rangle))$  |
   $((\langle \text{exprInSum} \rangle)_\sim (\langle \text{exprInSum} \rangle))$ 
<exprInFloorCeil> ::=
  ( $\langle \text{exprInFloorCeil} \rangle \langle \text{op} \rangle \langle \text{exprInFloorCeil} \rangle$ ) |  $\langle \text{iVar} \rangle$  |
   $\langle \text{iNumDenVar} \rangle$  |  $\text{sigma}(\langle \text{range} \rangle) (\langle \text{exprInSumFloorCeil} \rangle)$  |
   $((\langle \text{exprInFloorCeil} \rangle)^\sim (\langle \text{exprInFloorCeil} \rangle))$  |
   $((\langle \text{exprInFloorCeil} \rangle)_\sim (\langle \text{exprInFloorCeil} \rangle))$ 
<exprInSumFloorCeil> ::=
  ( $\langle \text{exprInSumFloorCeil} \rangle \langle \text{op} \rangle \langle \text{exprInSumFloorCeil} \rangle$ ) |
   $\langle \text{iVar} \rangle$  |  $\langle \text{kVar} \rangle$  |  $\langle \text{iNumDenVar} \rangle$  |  $\langle \text{kNumDenVar} \rangle$  |
   $((\langle \text{exprInSumFloorCeil} \rangle)^\sim (\langle \text{exprInSumFloorCeil} \rangle))$  |
   $((\langle \text{exprInSumFloorCeil} \rangle)_\sim (\langle \text{exprInSumFloorCeil} \rangle))$ 
<op> ::= + | -
<range> ::= forall_k_InLp_i | forall_k_InLep_i | forall_k_InHp_i |
  forall_k_InHep_i | forall_k
<iVar> ::=  $B_i$  |  $C_i$  |  $J_i$ 
<kVar> ::=  $C_k$ 
<iNumDenVar> ::=  $T_i$  |  $D_i$  (or in the recursive case ::=  $T_i$  |  $R_i$ )
<kNumDenVar> ::=  $T_k$  |  $J_k$ 

```

Proof-of-Concept:

- Evaluating recursive equations
 - ▶ Start with the no interference response time $J_i + C_i$
 - ▶ Repeatedly evaluate until the result converges
 - Or a max number of iterations is reached (100 in our experiments)
 - ▶ If result causes a divide-by-zero or overflow, then stop and use 0 or a large integer for the resulting value (results in poor fitness)

Proof-of-Concept: Experiments

- Verification vectors
 - ▶ Assume CAN bus with 11-bit message IDs
 - Messages from 1 to 8 data bytes
 - Period log-uniform in 5 to 500ms
 - Deadline in range 0.5 to 1.0 times period
 - Release jitter in range 0 to 0.5 times deadline
 - ▶ 100 verification vectors of 20 messages each with priorities in Deadline Monotonic order, utilisation from 50% to 97.5% (43 schedulable)
 - ▶ 25 verification vectors of 20 messages each in random priority order, utilisation from 25% to 35% (7 schedulable)
 - ▶ 10 verification vectors of 10 messages that highlight the flaw in the original CAN analysis, priorities in Deadline Monotonic order, utilisation from 95% to 99.5% (all unschedulable)

Proof-of-Concept: Experiments

■ Grammatical Evolution

- ▶ Implemented in EpochX open source genetic programming framework (v1.4.1)
 - Population size 1000
 - Generations 2000
 - Mutation rate 0.1
 - Fitness proportionate selection
- ▶ Each experiment was repeated 500 times

■ Assessment of evolved schedulability tests

- ▶ Against assessment vectors (similar to verification vectors but 10 times as many message sets)
- ▶ Manual checking for sufficiency (or not) via proof sketches and counterexamples

Proof-of-Concept: Experiments

▪ Expt 1: Baseline

- ▶ No recursion allowed by the grammar, precise indicative response times used, no corner case verification vectors

```
((Ji+sigma_(forall_k_InHp_i)
(((Ci)~(Ck*((Tk+(Ci+(Jk-(Ji-Di)))))-Tk)$Tk)))))+(Bi+Ci))
```

$$R_i = J_i + C_i + B_i +$$

$$\sum_{k \in hp(i)} \max \left(C_i, \left\lceil \frac{D_i - J_i + C_i + J_k}{T_k} \right\rceil C_k \right)$$

- ▶ Fitness 12624 (comparable to test **S2** – fitness 12038)
- ▶ Proven sufficient (sketch proof in appendix of paper)

Proof-of-Concept: Experiments

Expt 2: Adding recursion

- ▶ Recursion allowed by the grammar, precise indicative response times used, no corner case verification vectors

```

(((Ji+(Bi+(Ci+sigma_(forall_k_InHp_i)(Ck))))~(Bi))
+sigma_(forall_k_InHp_i)(Ck*((Ri-(Ji-Jk))
-(((Tk)~(Ck))~((Jk)~(Bi+((Tk-Ji)-Ck))))))$Tk))

```

$$R_i = J_i + C_i + B_i + \sum_{k \in hp(i)} \left\lceil \frac{R_i - J_i + J_k - \max(0, B_i - J_i - C_k)}{T_k} \right\rceil C_k$$

- ▶ Fitness 3084 (improves over test **S2** – fitness 12038, but still some way off test **S1** – fitness 1702)
- ▶ Proven not sufficient (counter example in appendix of paper)
 - Can sometimes produce optimistic results ' $B_i - C_k > C_i - \tau_{bit}$ ' i.e. when the blocking factor is more than the transmission time of the message or interest plus that of a higher priority message

Proof-of-Concept: Experiments

▪ Expt 3: Adding Corner Cases

- ▶ Recursion allowed by the grammar, precise indicative response times used, corner case verification vectors highlighting the flaw in test **F1**

```
(Bi+(Ci+(Ji+sigma_(forall_k_InHp_i)
(Ck*((( ((( (Ri)_ (Tk))) ~ (Ck)))_ (Jk)) + (Ri-Ji) $Tk))))))
```

$$R_i = J_i + C_i + B_i + \sum_{k \in hp(i)} \left\lceil \frac{R_i - J_i + \min(\max(R_i, C_k), J_k)}{T_k} \right\rceil C_k$$

- ▶ Fitness 3096 (similar to Expt 2, but still some way off test **S1** – fitness 1702)
- ▶ Proven not sufficient (counter example in appendix of paper)
 - Can sometimes produce optimistic results when the release jitter of a higher priority message is very large permitting back-to-back interference (case not covered in the verification vectors)

Proof-of-Concept: Experiments

▪ Expt 4: Adding Corner Cases

- ▶ Recursion allowed by the grammar, approximate indicative response times used from 0.8 to 1.0 times the exact values, corner case verification vectors highlighting the flaw in test **F1**

```
((Ji+(Bi+Ci))+sigma_(forall_k_InHp_i)
(Ck*(((Ci+Ri)+Jk)-Ji)$Tk))
```

$$R_i = J_i + C_i + B_i + \sum_{k \in hp(i)} \left[\frac{R_i - J_i + C_i + J_k}{T_k} \right] C_k$$

- ▶ Fitness 4710 (somewhat worse than Expts 2 and 3)
- ▶ Proven sufficient (sketch proof in appendix of paper)

Proof-of-Concept: Experiments

▪ Expt 5: Random search

- ▶ Recursion allowed by the grammar, exact indicative response times, corner case verification vectors highlighting the flaw in test **F1**
- ▶ Search not directed just tries different expressions at random and returns the one with the best fitness

```
((((( (((((( (Ji) ~ (Bi)) ~ (Bi*(Ci$Ri)))) ~ (Ci*(Ti$Ci)))) ~ (Ji))) ~ (Ji)))+sigma_(forall_k_InHp_i) (Ck))
```

$$R_i = \left[\frac{T_i}{C_i} \right] C_i + \sum_{k \in hp(i)} C_k$$

- ▶ Fitness 56477 (an order of magnitude worse than the evolutionary approach)
- ▶ Typically returns values greater than period so nearly all messages deemed unschedulable (useless as a schedulability test)

Lessons learnt #1

▪ Improving the grammar

▶ Dimensionality:

- Restricted use of ceiling, floor, and multiply to only appear in compound operators – ensures expressions are scale invariant and dimensionally correct

▶ Nesting restrictions:

- Constrained summation so it could not be nested – avoids very complex expressions

▶ Symbol restrictions:

- Actual response times can only be composed of multiples of C_i , C_k , J_i , B_i all other symbols can only be used in their multipliers

Lessons learnt #2

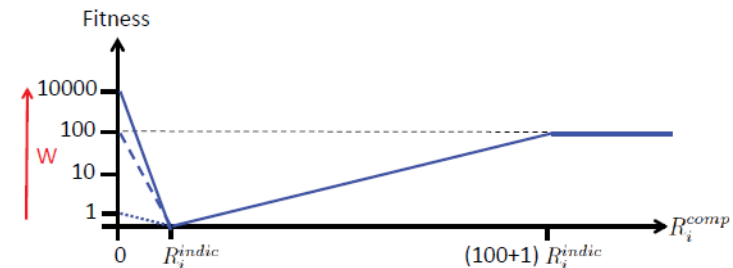
- Improving the verification vectors
 - ▶ **Correlations between parameters**
 - Had to be avoided as this could lead to inappropriate substitution of one parameter for another e.g. if deadlines were in DM order
 - ▶ **Range of values for variables:**
 - Small values for release jitter initially used meant that J_i could serve as an incorrect proxy for C_i or B_i
 - Expression from Expt 3 is not sufficient when jitter values are very large – points to need to improve the set of verification vectors
 - Transmission times had to take a range of values (not all the same) otherwise C_i , J_i , and B_i would not be distinguished - similarly if implicit deadlines were used D_i , and T_i could not be distinguished

Lessons learnt #3

▪ Tuning the Grammatical Evolution

▶ Fitness function

- Vitally important needs to heavily penalise under-approximation of response times, but a heavy penalty in early generations can be counter productive hence the variable weighting factor used



▶ Number of generations

- Explored 500, 1000, 2000, 5000 no significant improvement after 2000

▶ Escaping local minima

- Grammatical Evolution is a trade off between exploration of the vast search space and exploitation of good candidates
- Small changes to an expression can result in large changes to fitness that cause difficulties in escaping local minima
- Mitigated by repeating experiments a large number of times (500) and taking the best overall results

Lessons learnt #4

▪ Improving the implementation

▶ Parser

- Standard EpochX parser is capable of handling arbitrary expressions, which makes it slow when all that is needed is to parse mathematical expressions
- Implementing our own parser (two pages of Java code) improved the overall runtime by a factor of 100

▶ Parallel execution

- Using a high performance compute cluster enabled evolving the 500 populations for each of our experiments in parallel

Conclusions

- Research Contributions from the Proof-of-Concept
 - ▶ **Formulation assistance** using Grammatical Evolution is a viable approach that can provide interesting candidate equations for schedulability tests
 - ▶ **Candidate equation quality** similar to known sufficient tests in terms of performance w.r.t. the assessment vectors
 - ▶ **The burden of proof (of sufficiency) remains with the researcher**
 - ▶ Formulation assistance is only assistance **not** automation
 - Of the four experiments the best results from two were sufficient tests, while the other two were not
 - ▶ Some performance degradation when approximated indicative response times are used (as would be the case when no exact test is known and the worst-case cannot be simulated) but still resulted in useful schedulability tests

Future Research

▪ Directions

▶ Improving the basic method

- Explore an **Island Based Approach** that enables re-combination and evolution of the best solutions found in different island populations evolved in parallel – aims to avoid evolutionary dead-ends (local minima)

▶ Unsolved schedulability analysis problems

- Apply the formulation assistant concept to unsolved schedulability analysis problems, in the area of Network-on-Chip, using simulation as a means of determining indicative response times

▶ Co-evolution of verification vectors

- Important to avoid candidate equations being wrongly classified as sufficient – this is difficult as simulation can only provide necessary tests
- Possible improvement is to co-evolve verification vectors to better identify corner cases

▪ Formulation assistance not automation

- ▶ Researchers still need to interpret the expressions derived, refine them and seek to prove their correctness by other means (e.g. proof assistance)



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