

Necessary Feasibility Analysis for Mixed-Criticality Task Systems on Uniprocessor

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Highlights

The paper provides
tight necessary feasibility tests
for mixed-criticality (MC) systems on uniprocessor

the first study that yields non-trivial results
for MC necessary feasibility:

- Reducing a set of MC task sets whose feasibility is unknown by existing studies
- Identifying unique issues of developing necessary feasibility tests for MC systems

“Feasibility” of timing guarantees

- Is it possible to successfully schedule every instance of all tasks without missing any deadlines?

“Feasibility” of timing guarantees

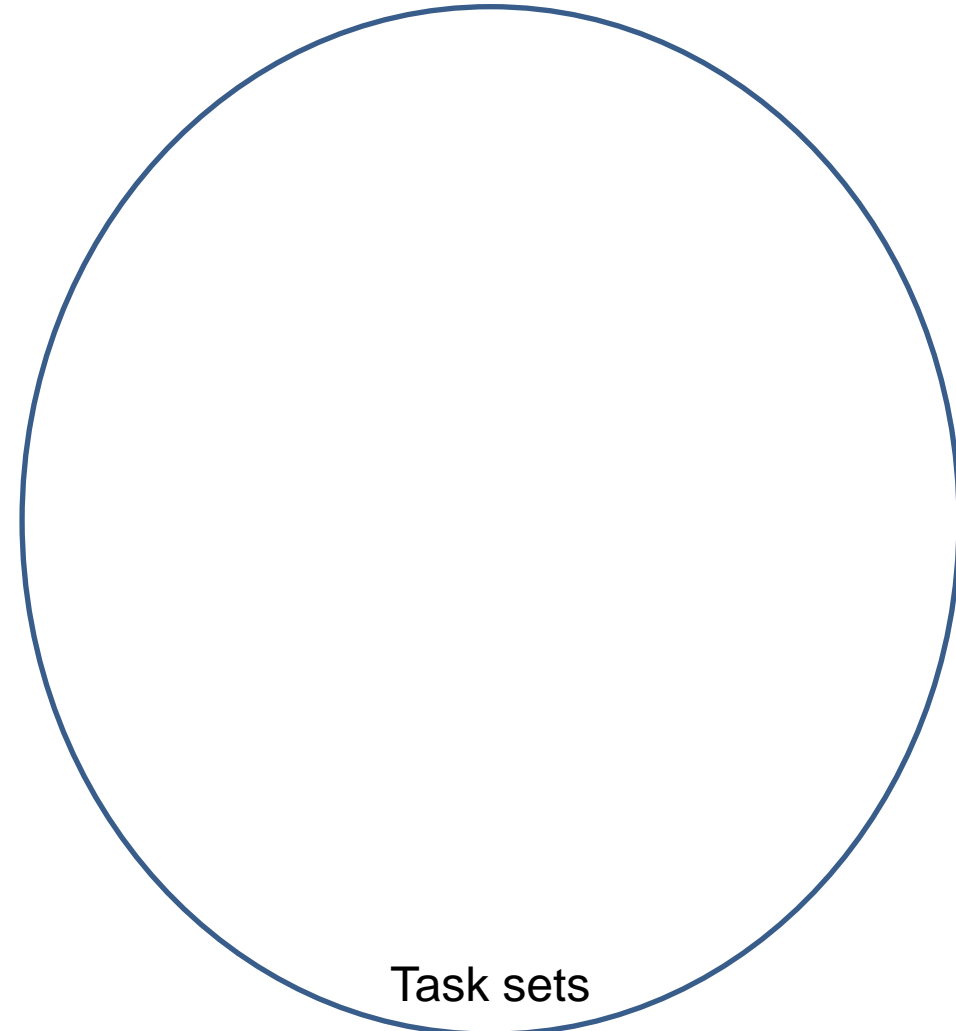
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- Two directions
 - Addressing sufficient feasibility
(Finding “Yes” answers)
 - Addressing necessary feasibility
(Finding “No” answers)

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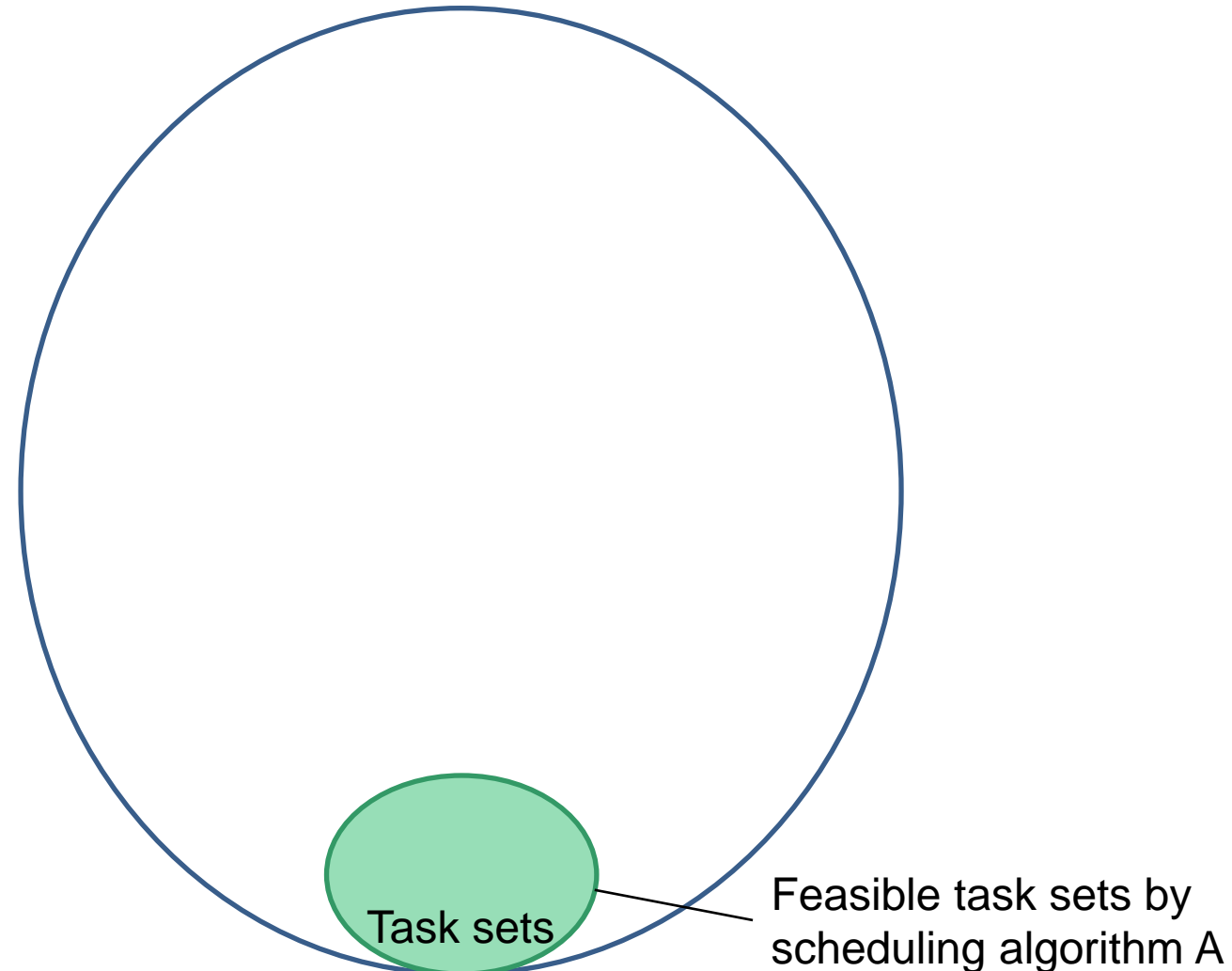
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Task sets

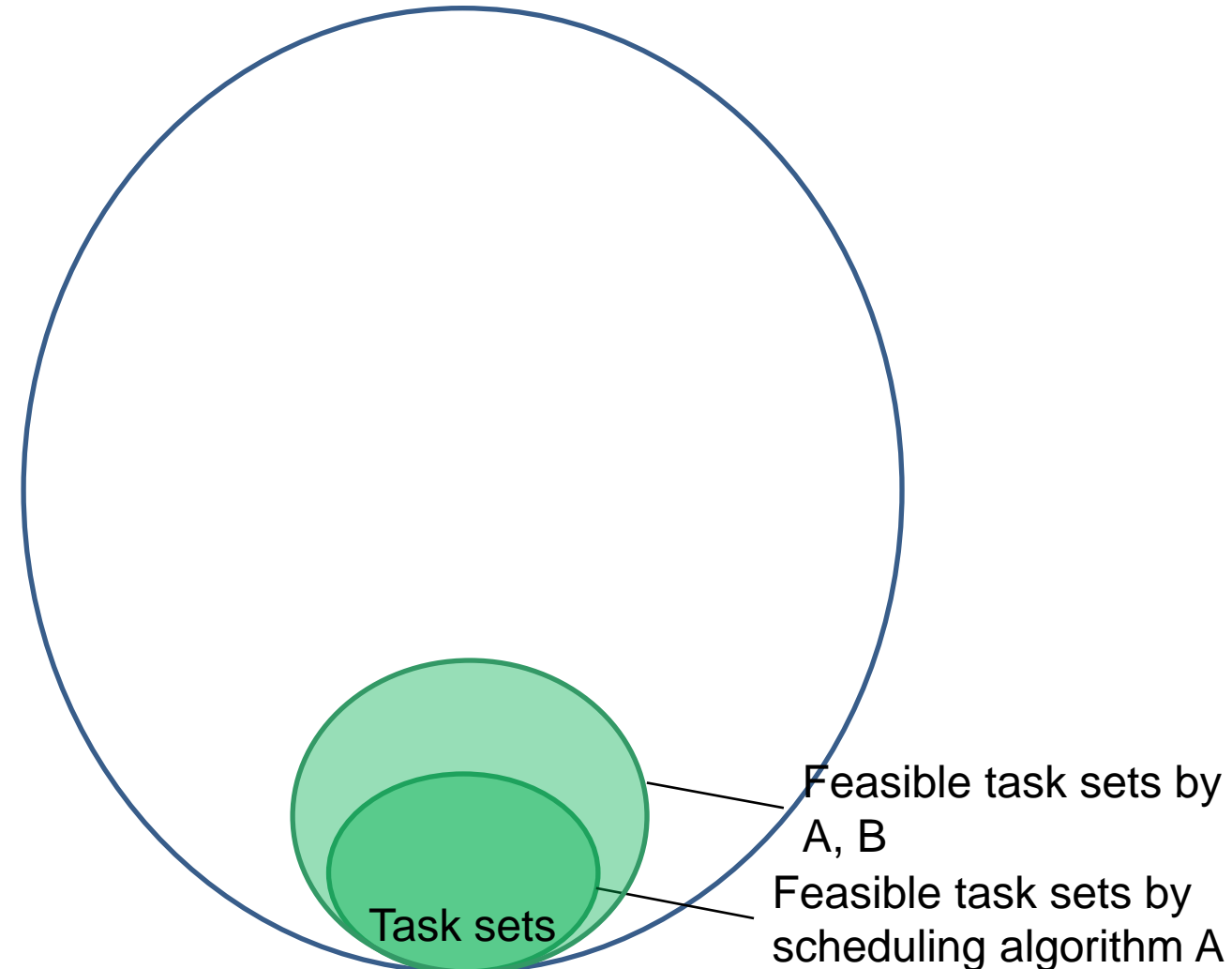
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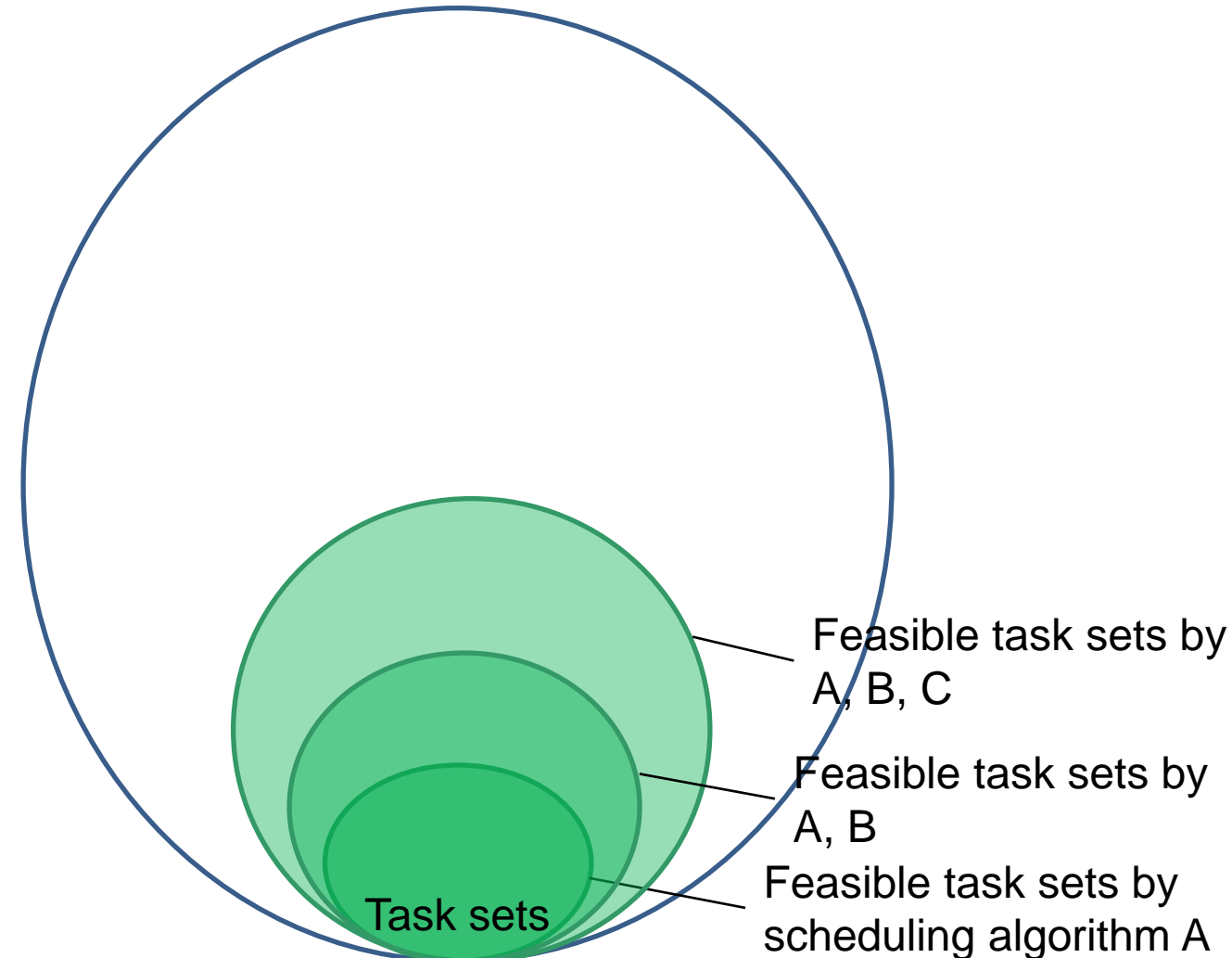
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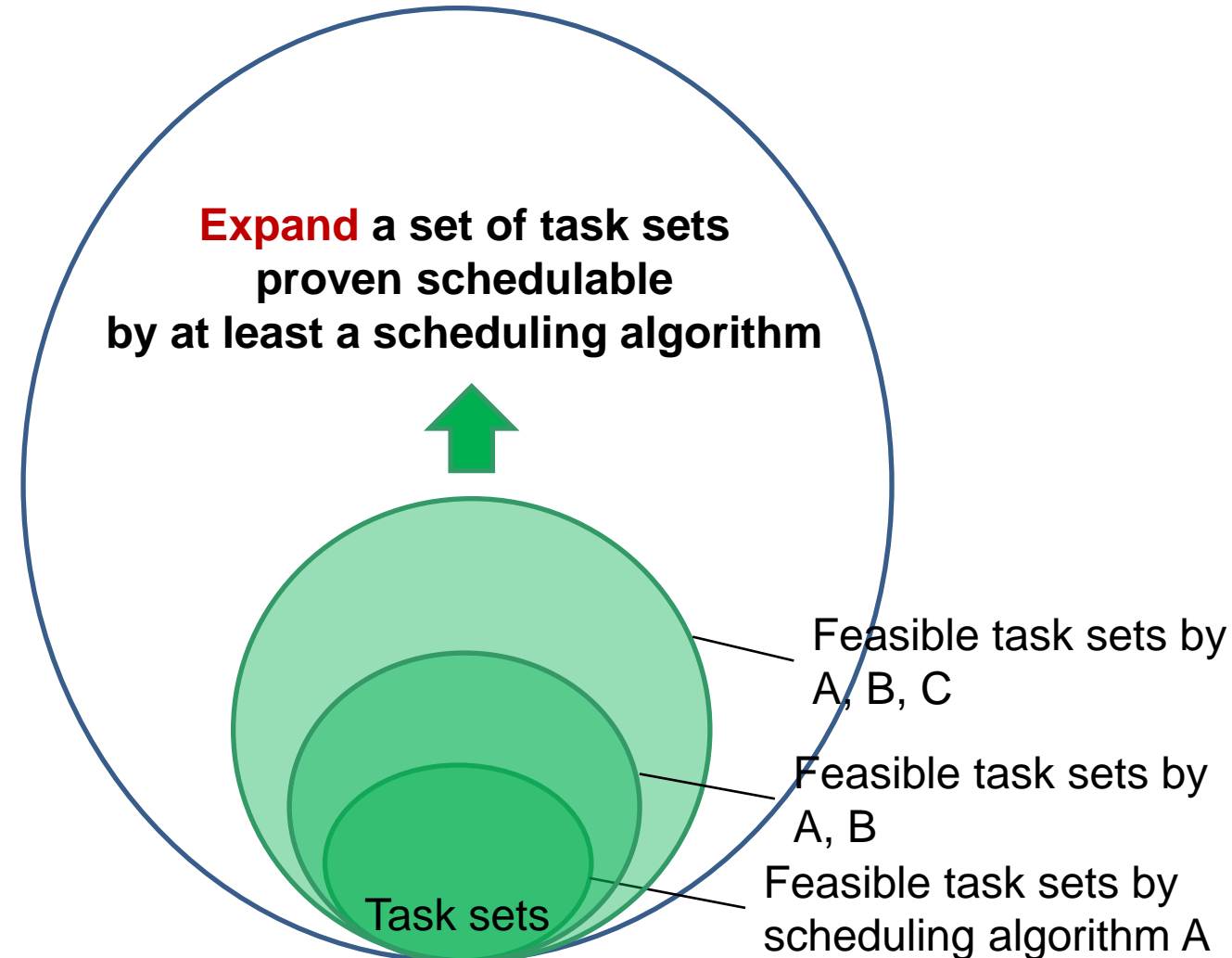
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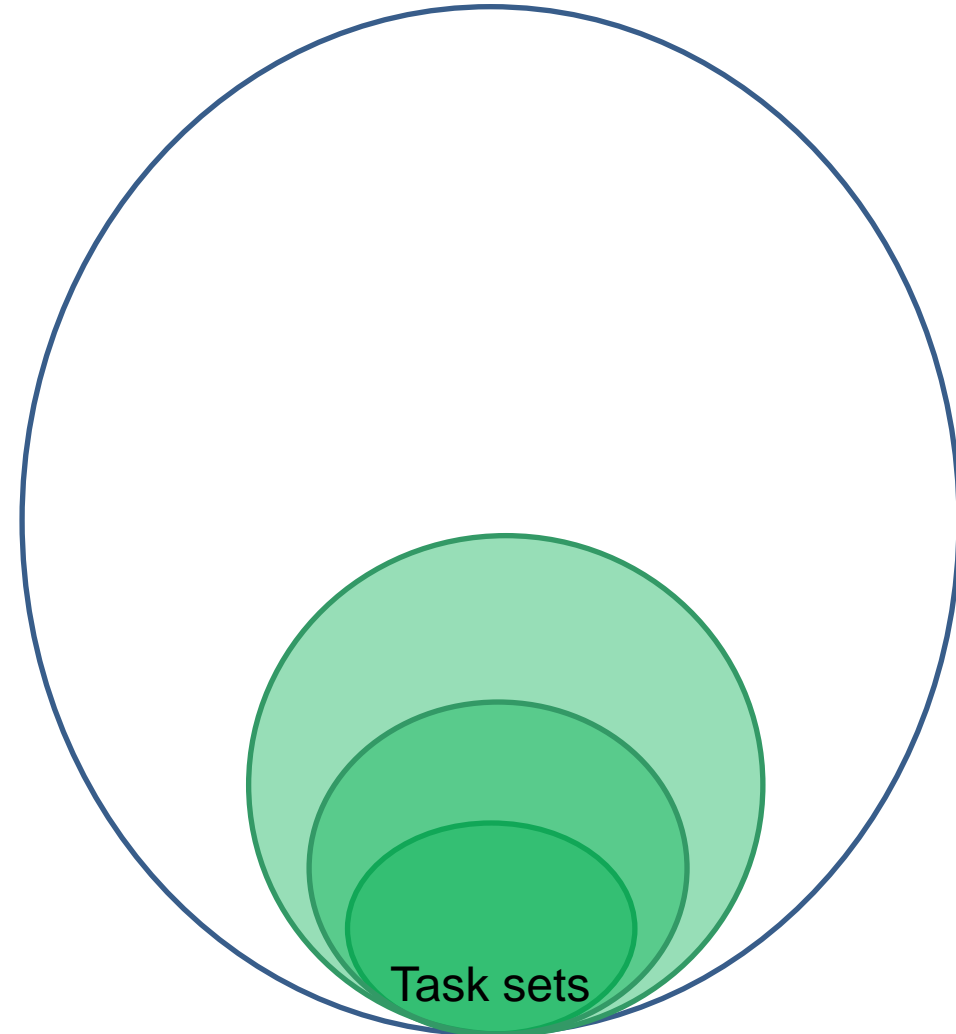
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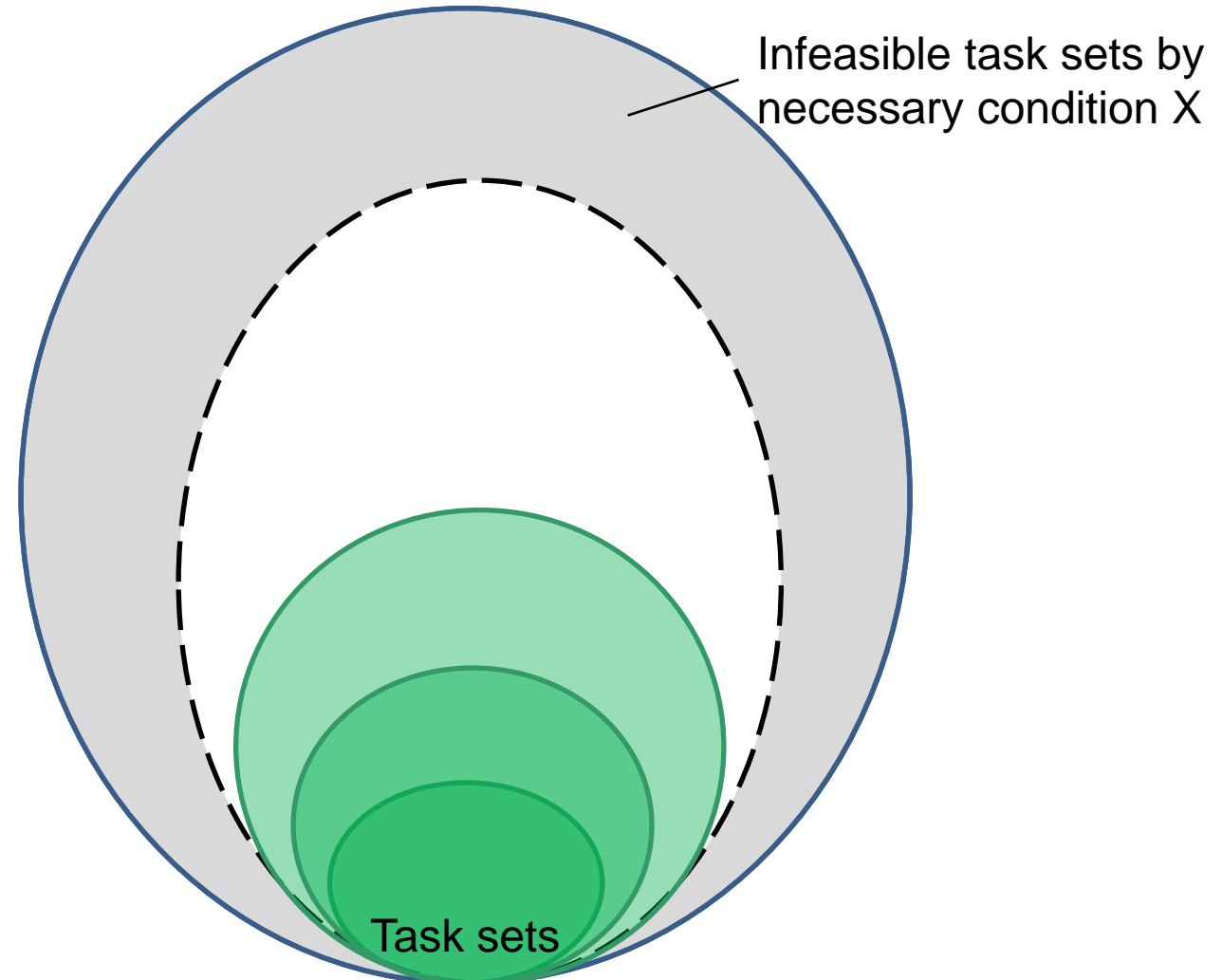
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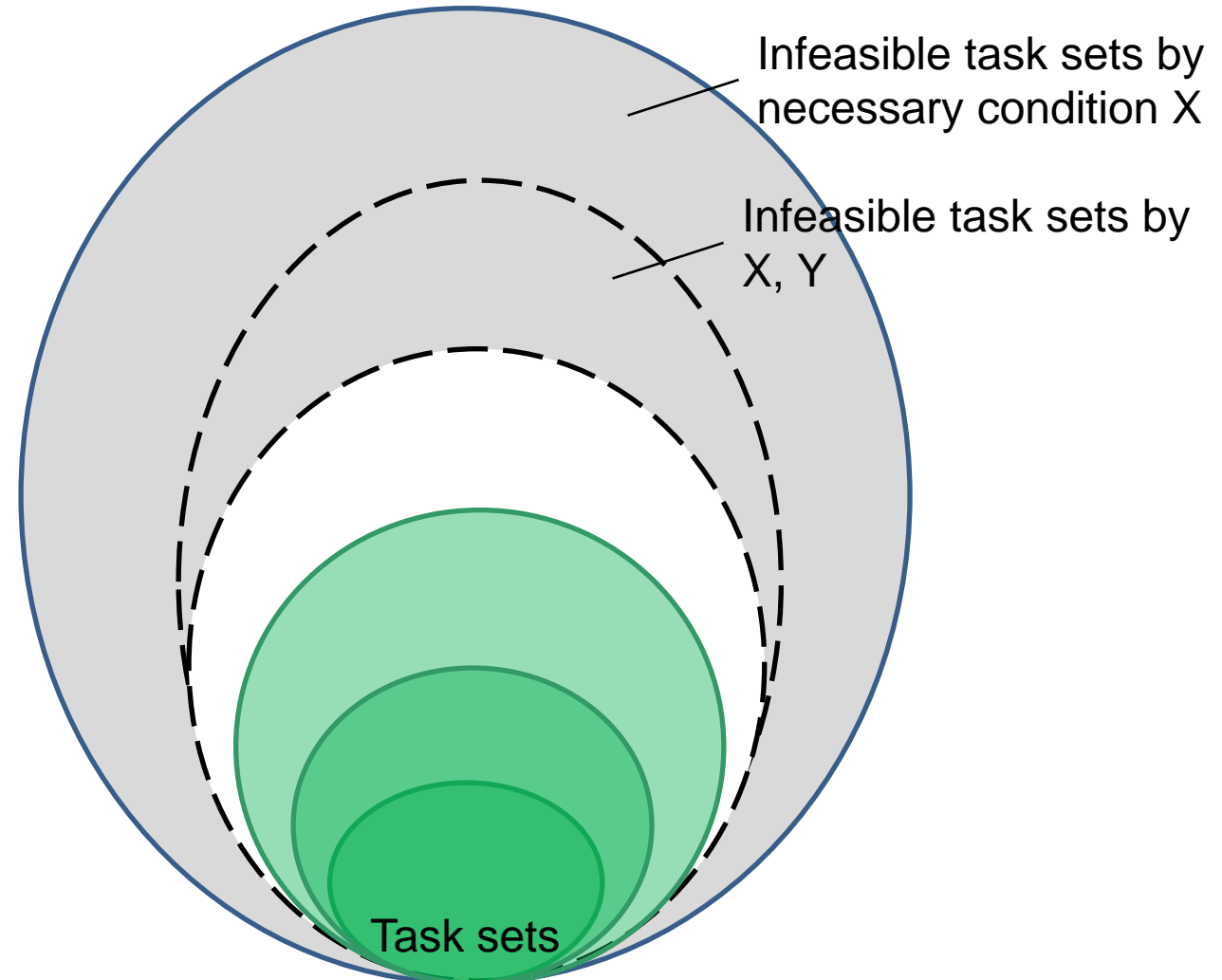
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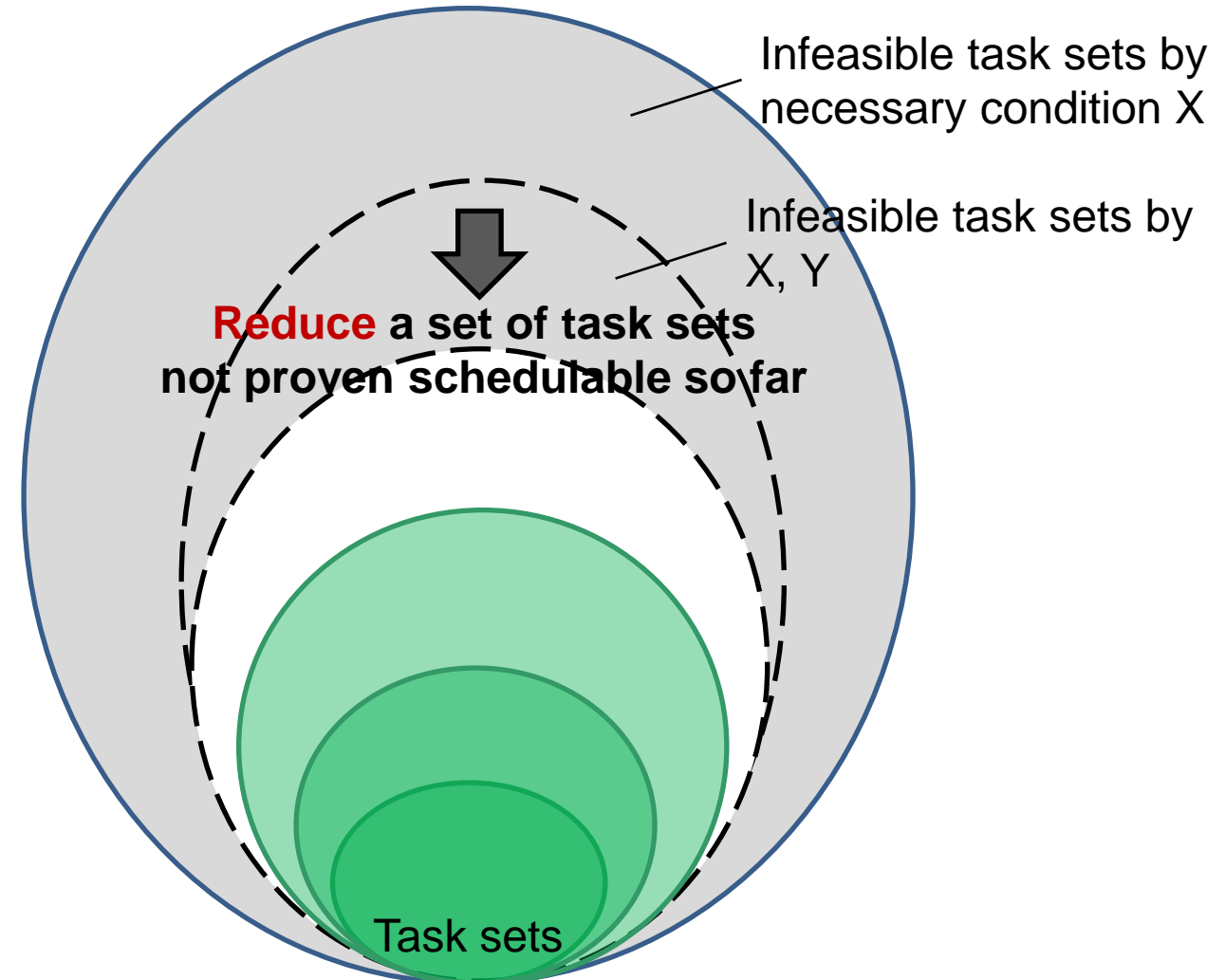
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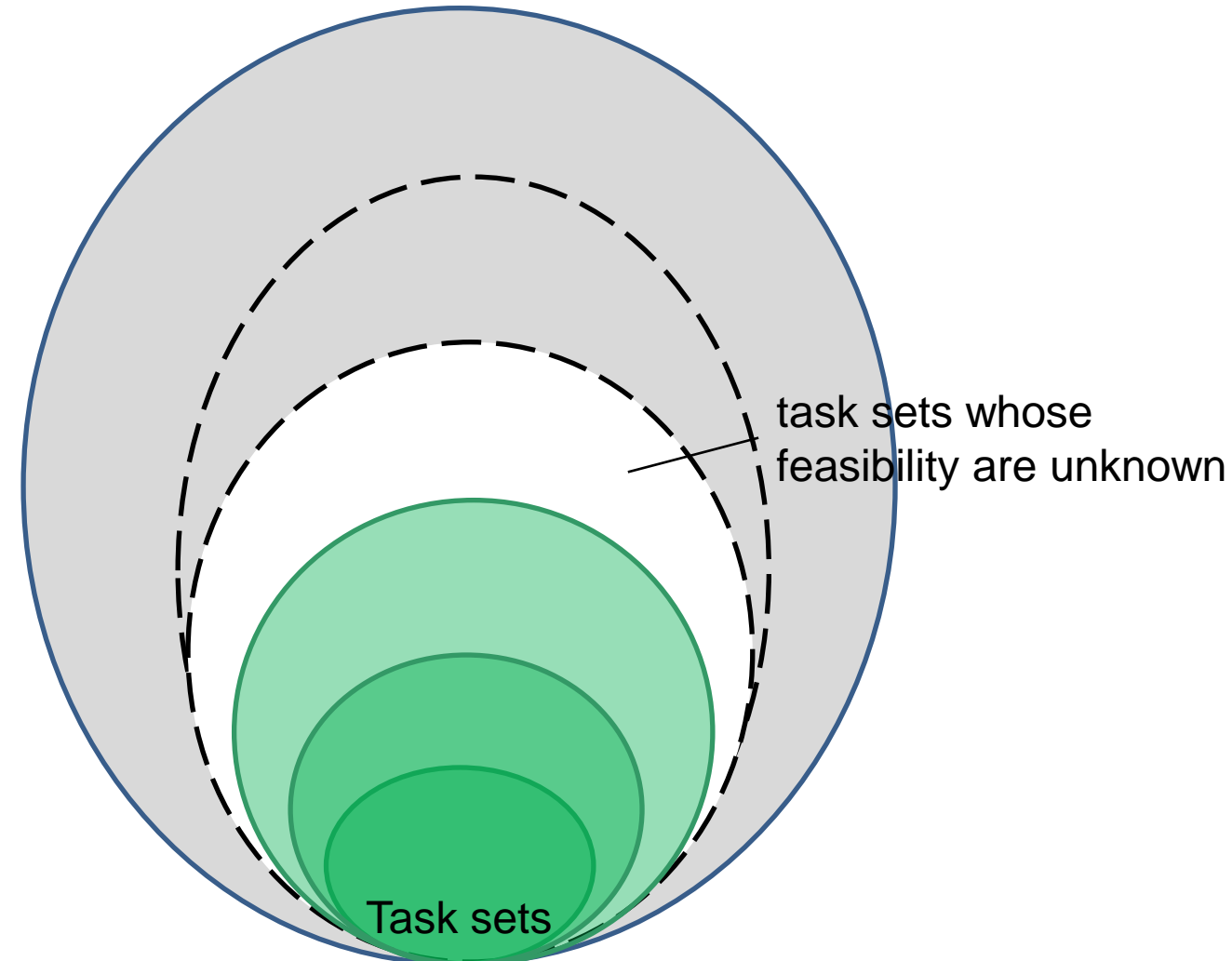


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State of the Art

Single-Criticality Task Systems

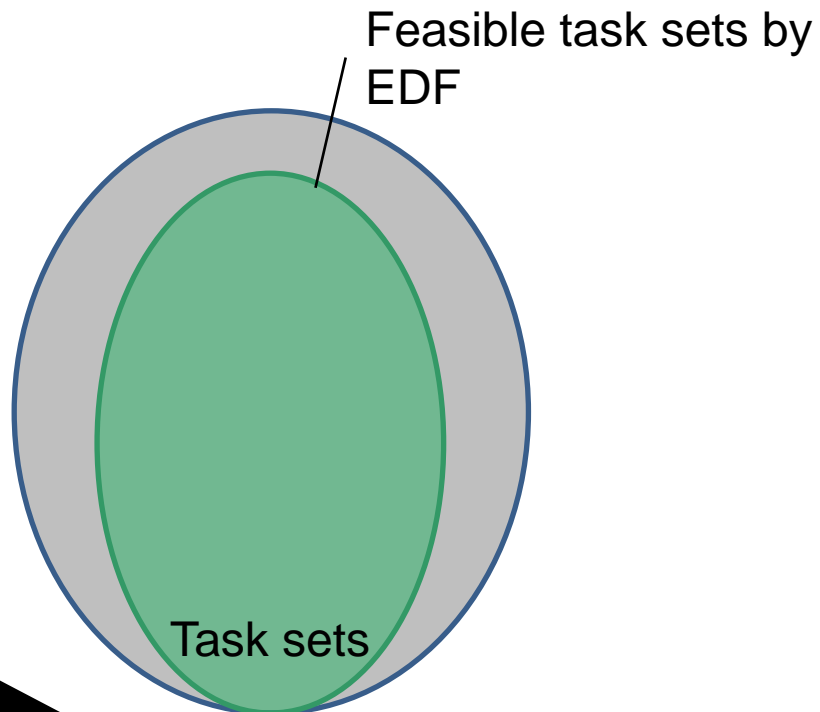
Mixed-Criticality Task Systems



State of the Art

Single-Criticality Task Systems

- + Exact feasibility analysis
- + Optimal scheduling algorithm (EDF)

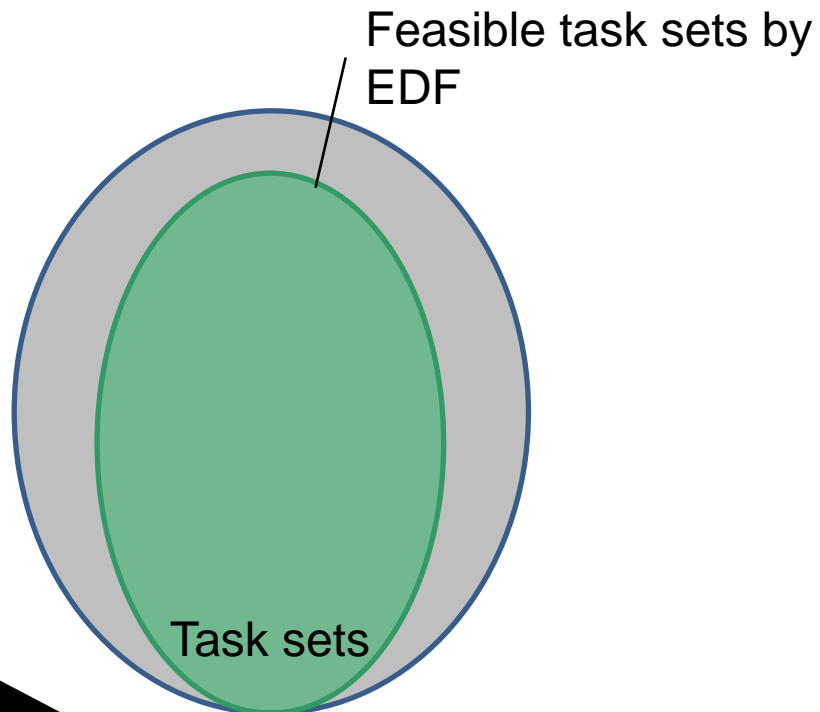


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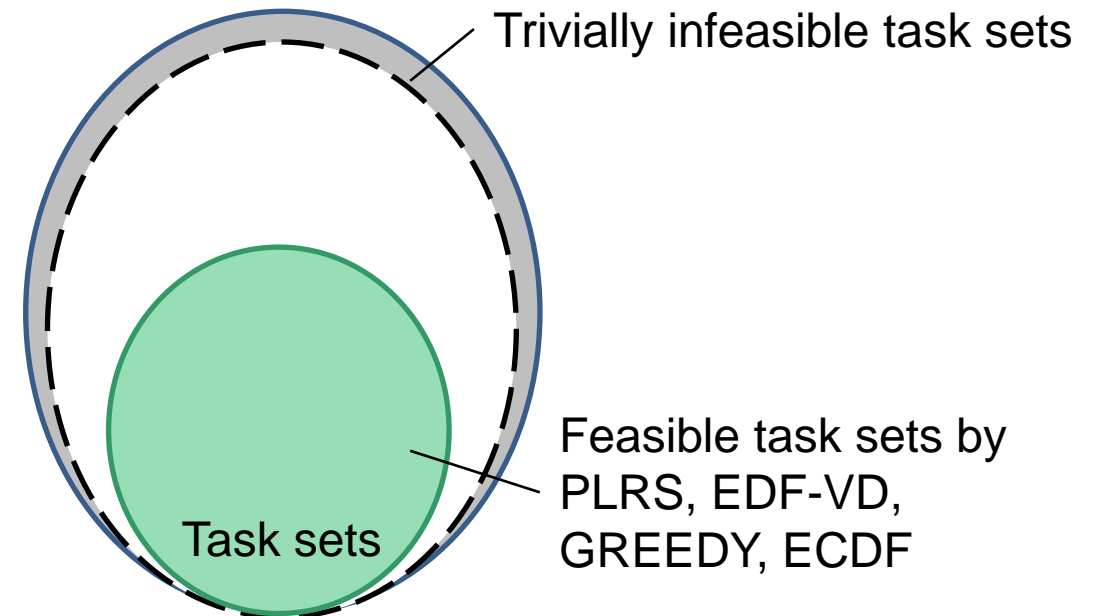
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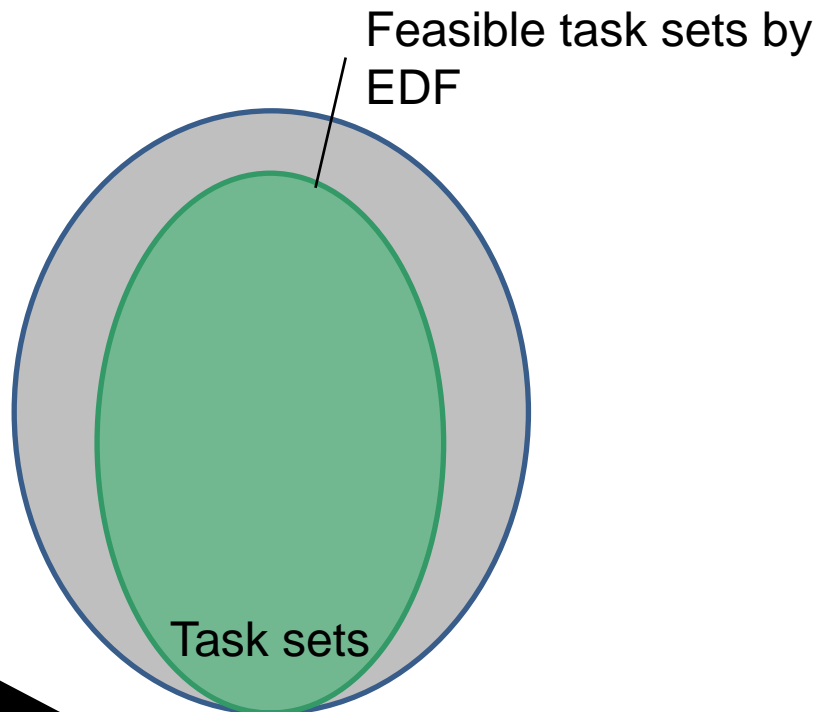
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- No optimal scheduling algorithm
- Only a few existing necessary feasibility condition



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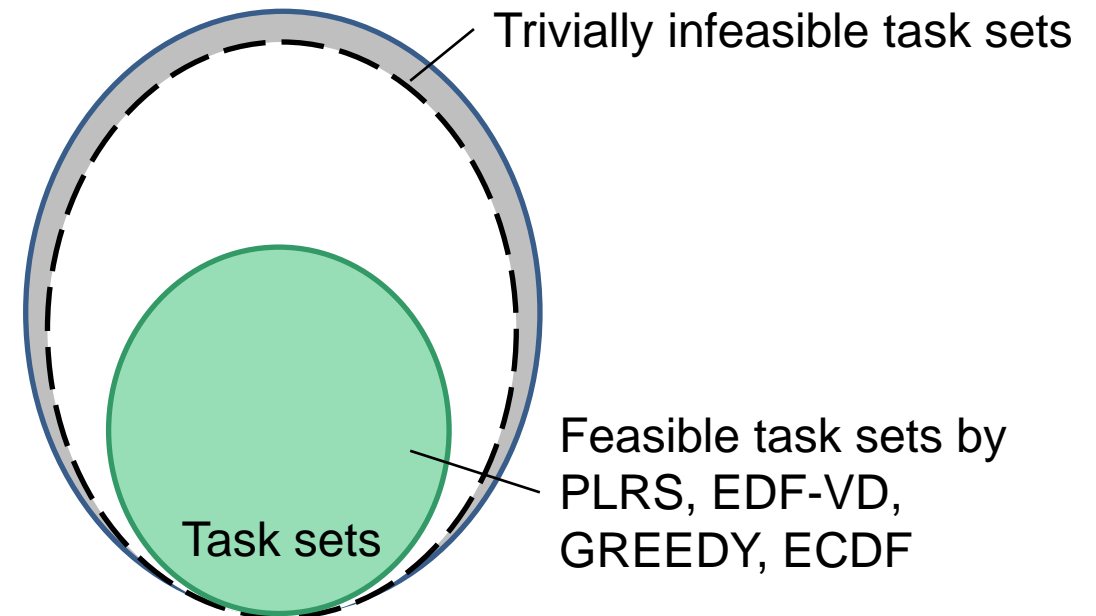
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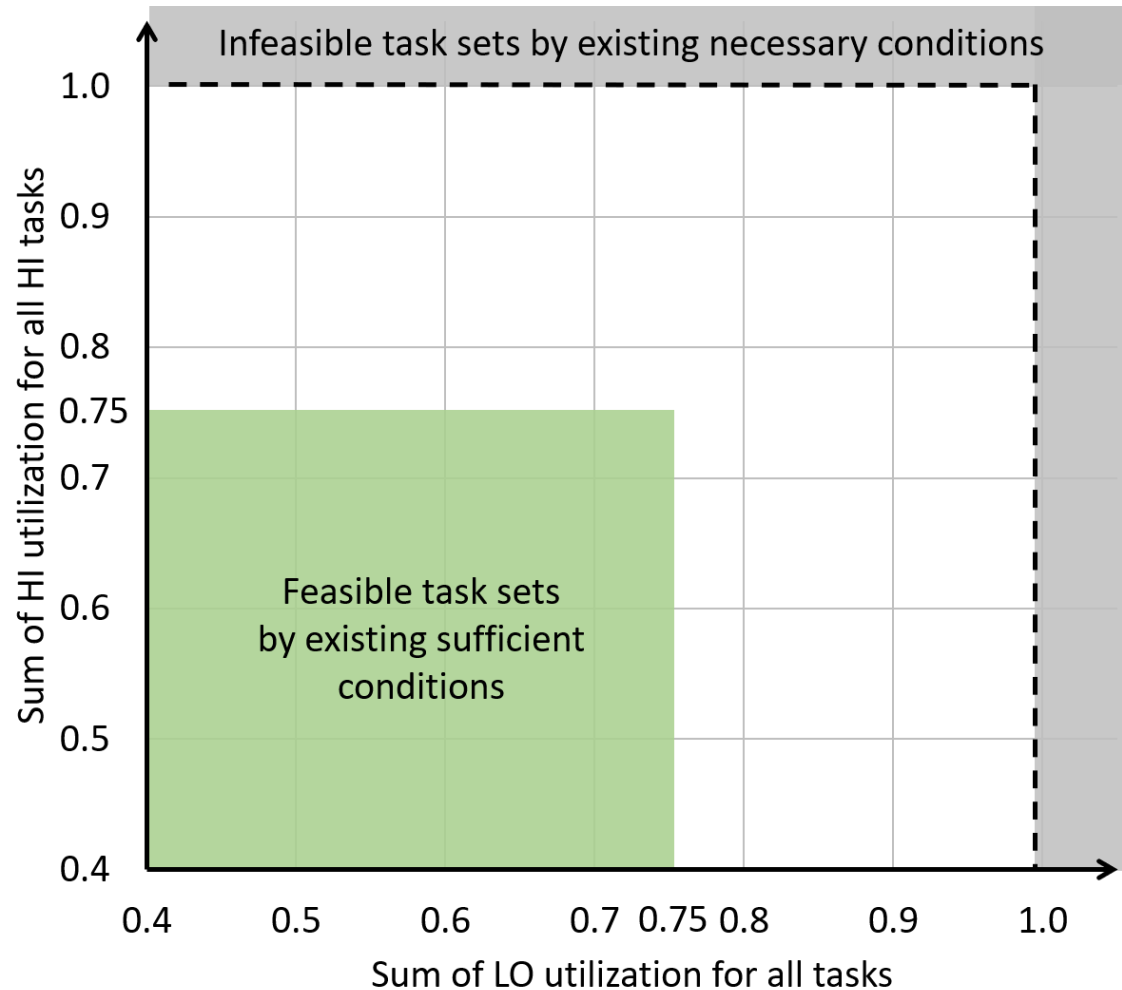
Still huge gap between the task sets proven feasible and that proven infeasible



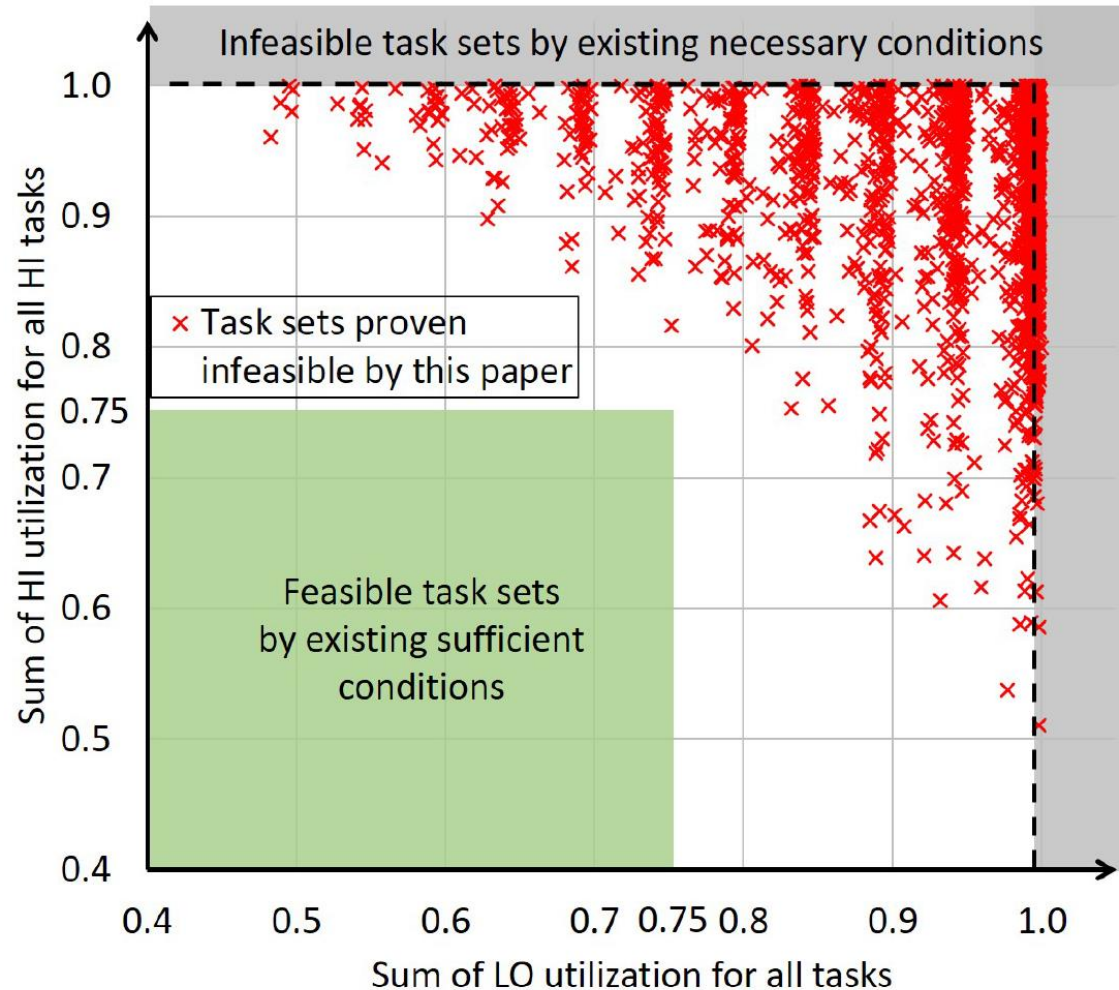
Goal

- Develop **necessary feasibility tests** that cover a broader range of infeasible MC task sets on a uniprocessor
 - Determining MC-feasibility for dual-criticality task systems is known to be NP-hard

Contributions of This Work

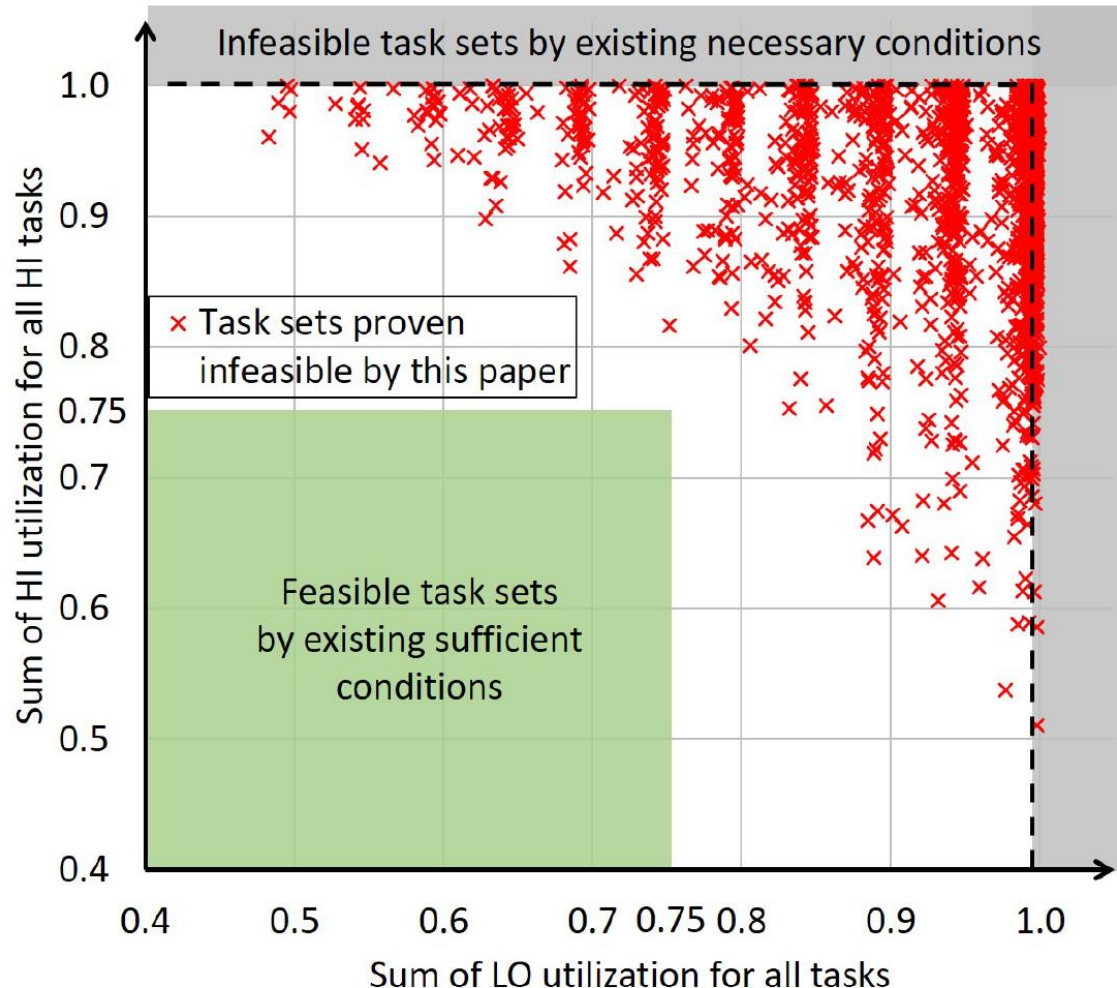


Contributions of This Work



The first study that yields non-trivial results
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Contributions of This Work



The first study that yields non-trivial results for **MC necessary feasibility**

- ➔ Explore **unique issues** specific to MC task systems for developing necessary feasibility tests
- ➔ Identify **new challenges** posed by such unique issues of MC task systems
- ➔ Establish **foundations** of necessary feasibility tests for MC task systems

System Model

- Dual-criticality systems (Vestal's task model)
 - **Task** $\tau_i = (T_i, \chi_i, C_i^{LO}, C_i^{HI}, D_i)$, where
 - $\chi_i \in \{LO, HI\}$;
 - LO – low-critical task, HI – high-critical task
 - C_i^{LO} : LO WCET, C_i^{HI} : HI WCET
 - for LC task $C_i^{LO} = C_i^{HI}$ and for HC task $C_i^{LO} \leq C_i^{HI}$
 - **Job** $J_i^q = (r_i^q, \gamma_i^q)$, where
 - r_i^q : the release time of the job
 - $\gamma_i^q \in (0, C_i^{HI}]$: the execution requirement
 - **Scenario** for a given task set τ
 - A collection of release times and execution requirements of jobs invoked by tasks in τ

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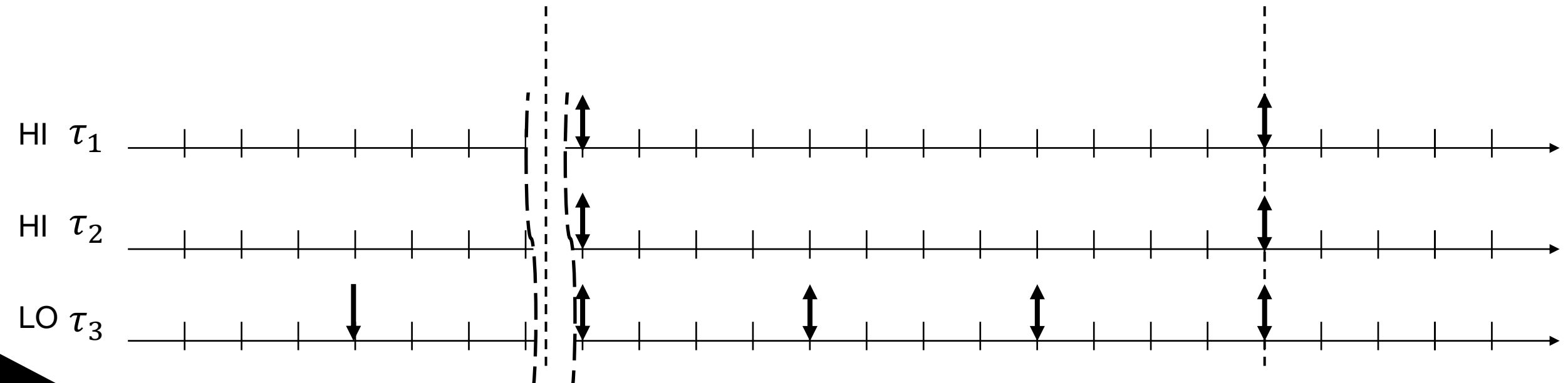
MC-feasible (MC-infeasible)

- If *every* scenario is feasible (If there exists *at least one* scenario that is not feasible)

- **Feasible scenario**
 - If there exists a schedule that satisfies
 - i) **every** job finishes its execution time before its deadline when exhibiting the **LO behavior**
 - ii) **every HI** job finishes its execution time before its deadline when exhibiting the **HI behavior**

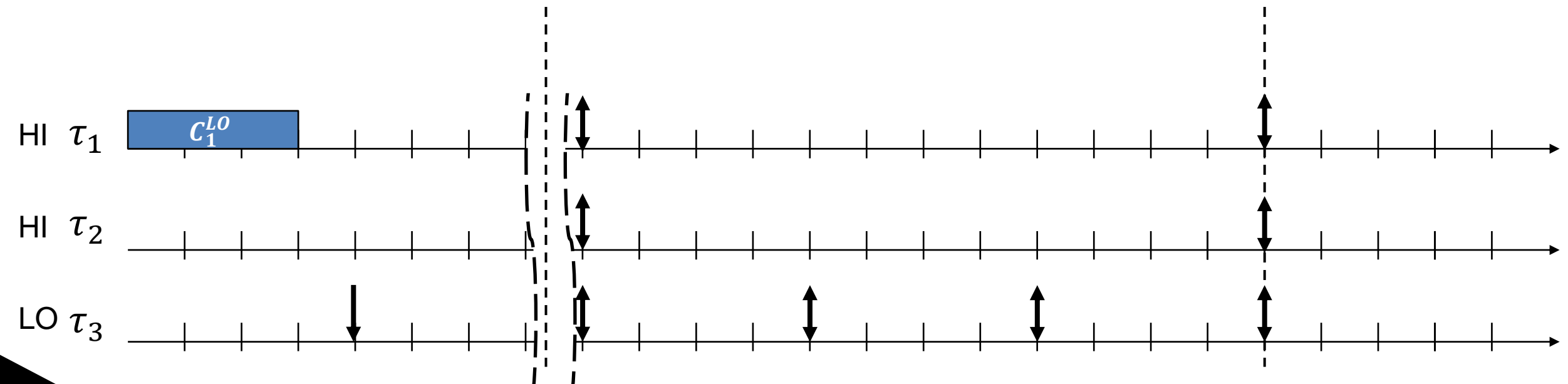
Unique Characteristics of MC Task Systems

- Existence of the mode change



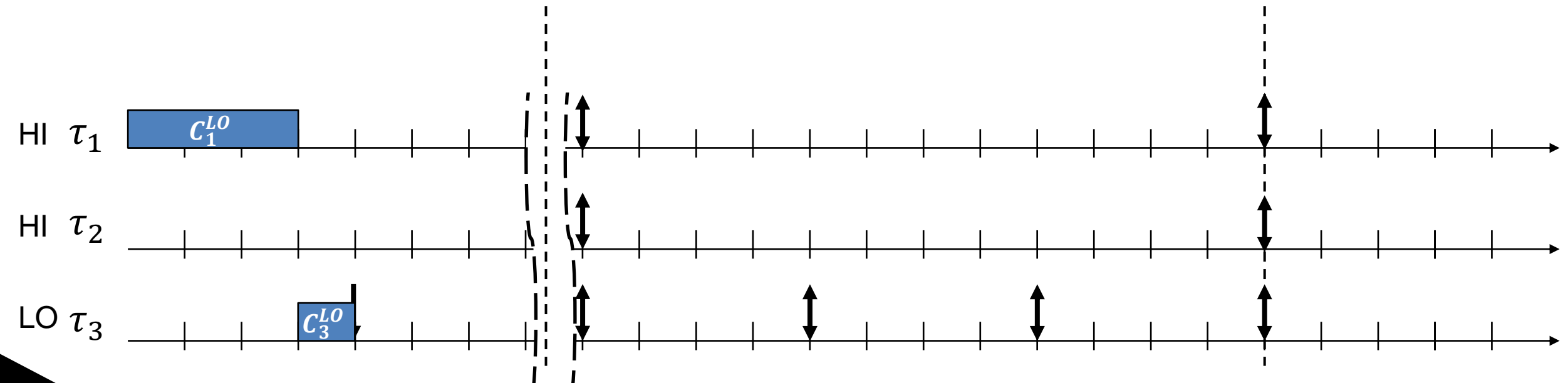
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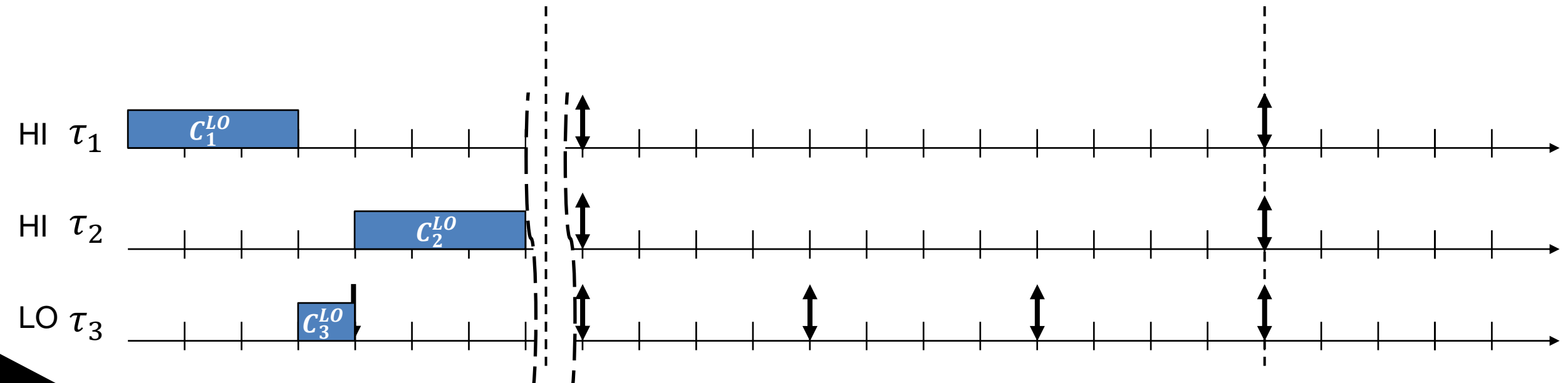
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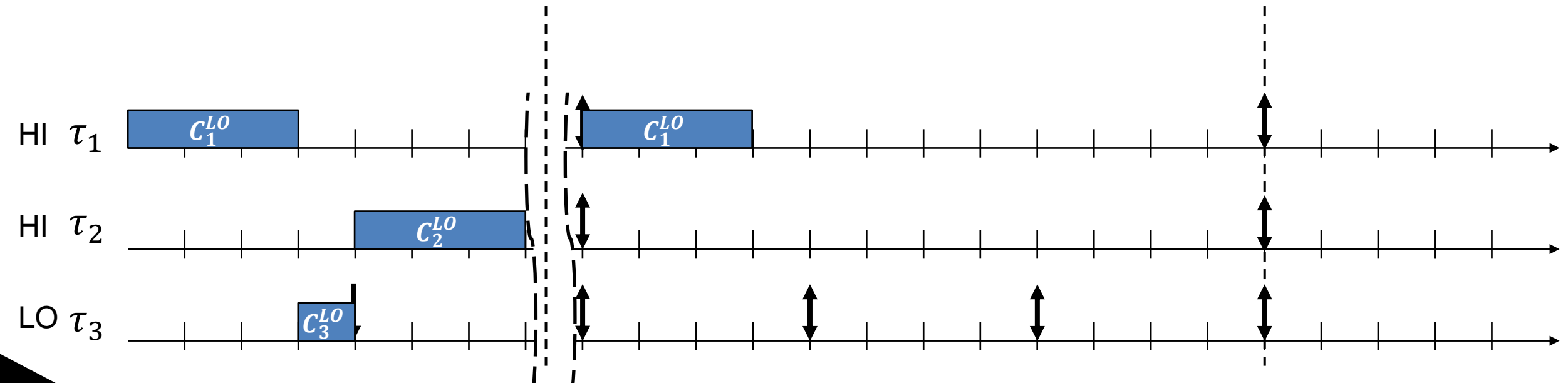
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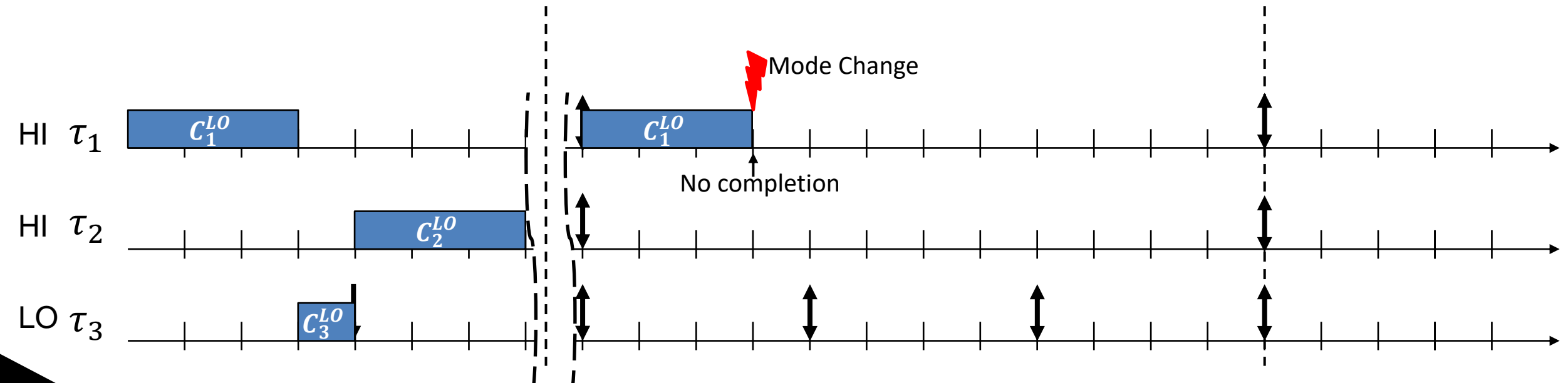
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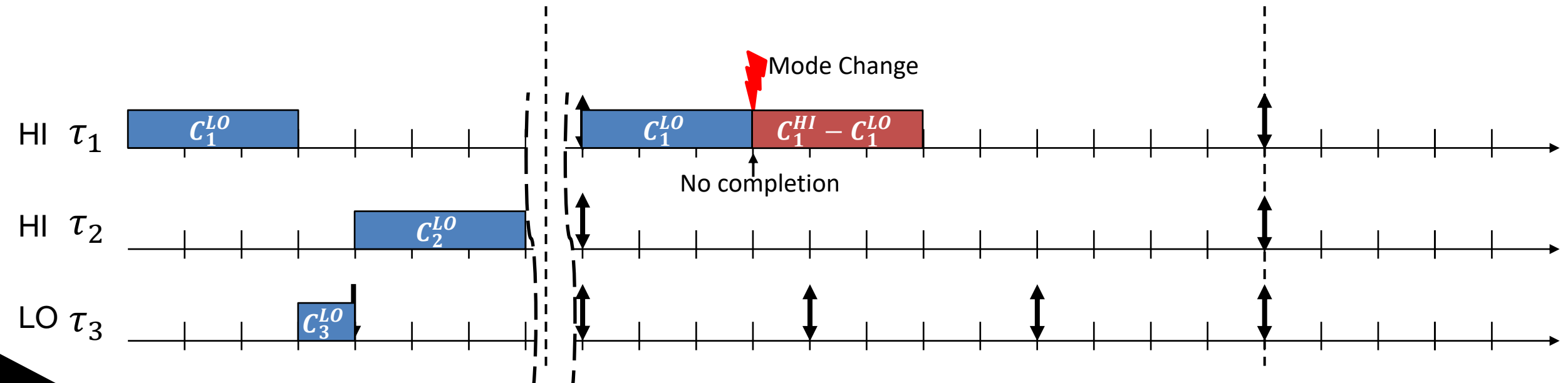
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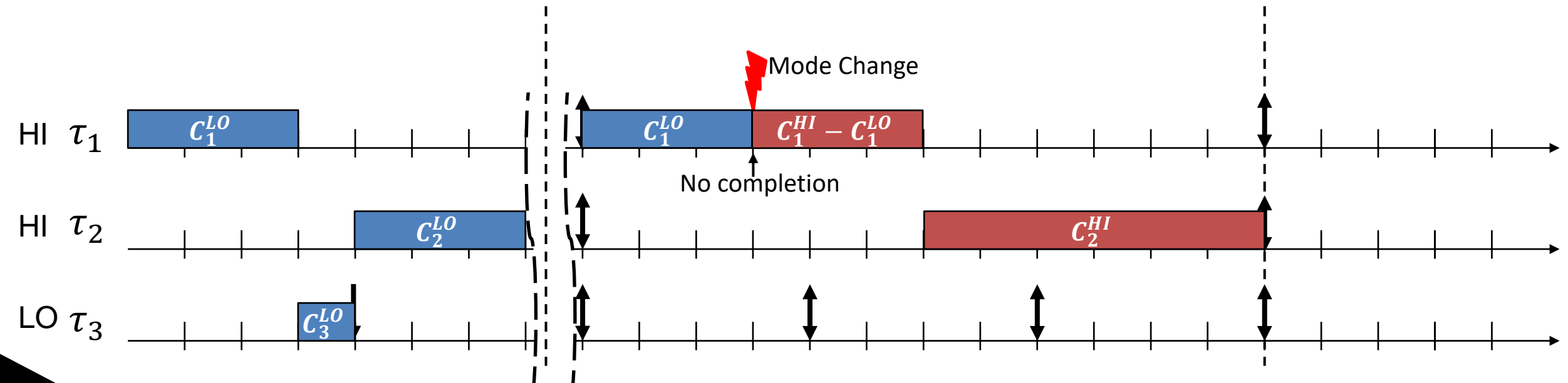
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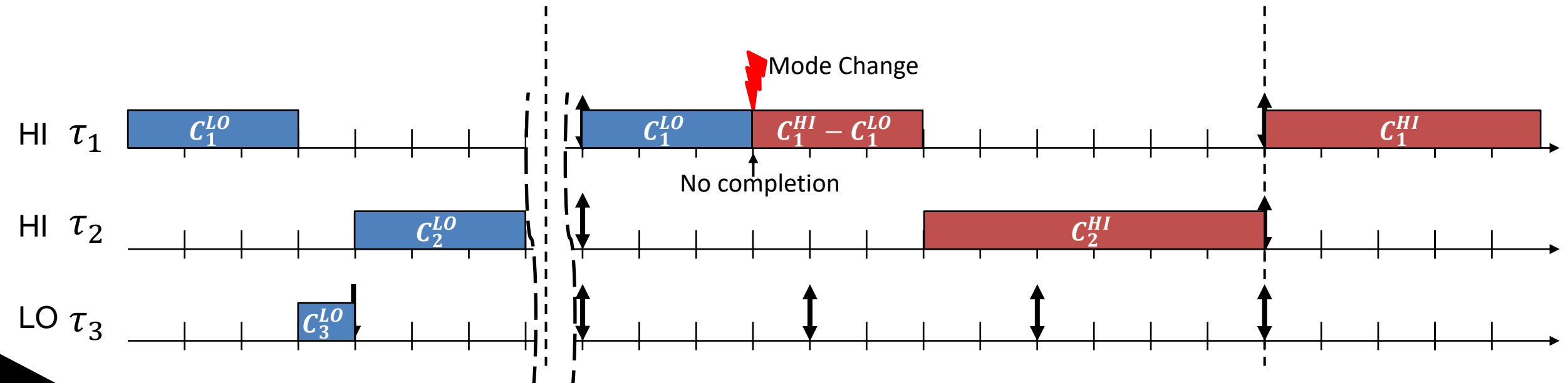
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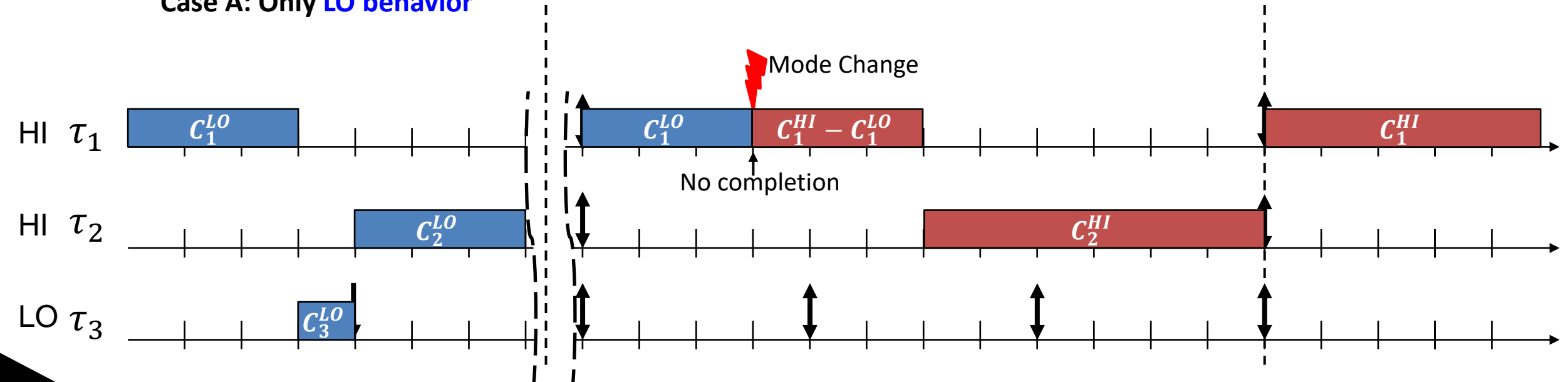
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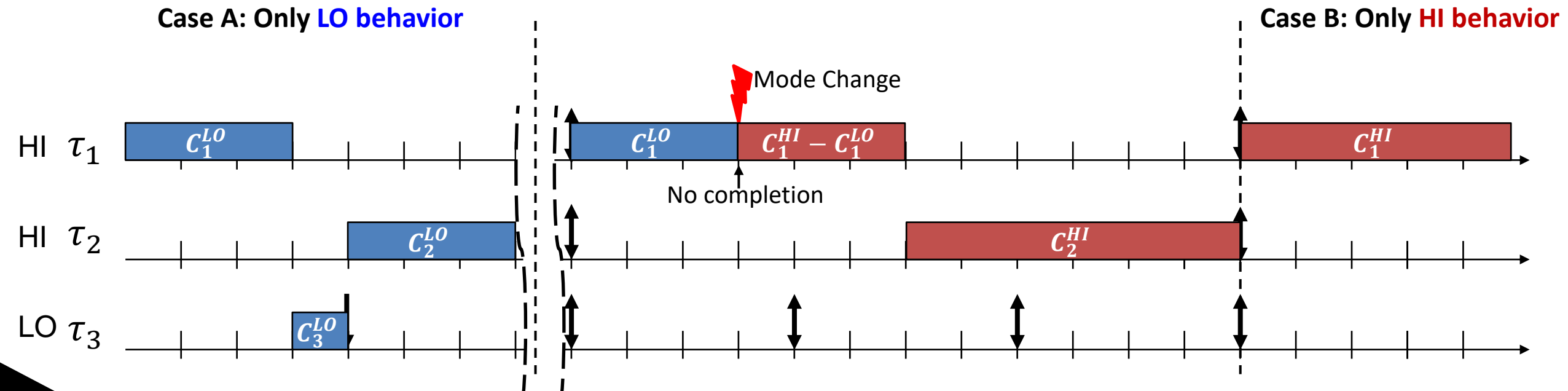
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Case A: Only LO behavior



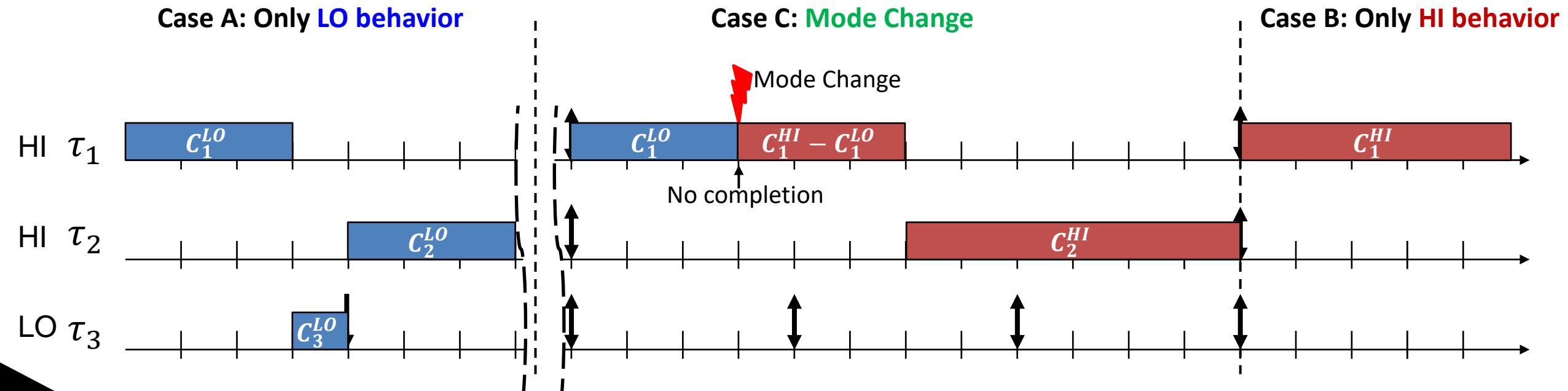
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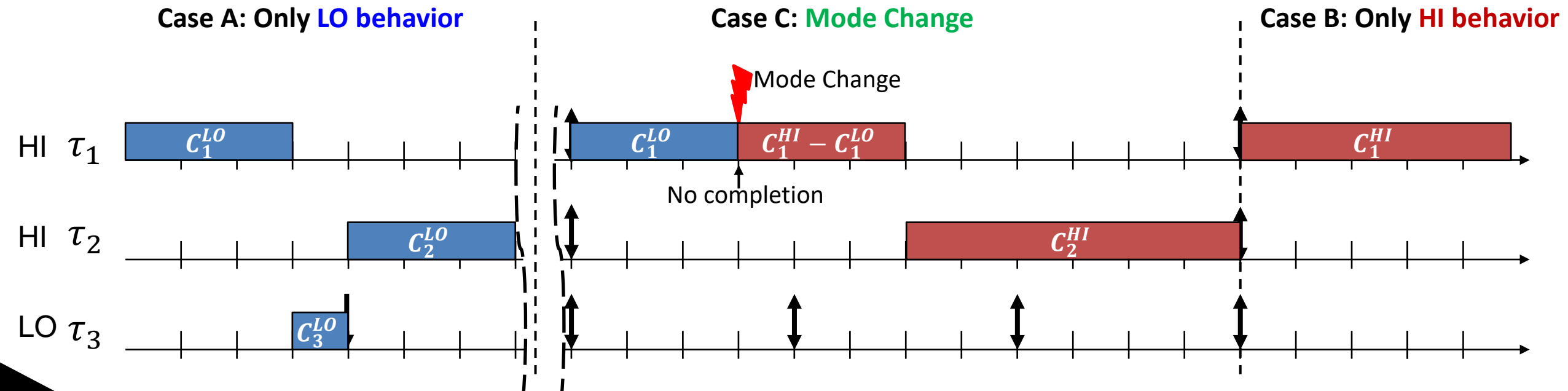
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Unique Characteristics of MC Task Systems

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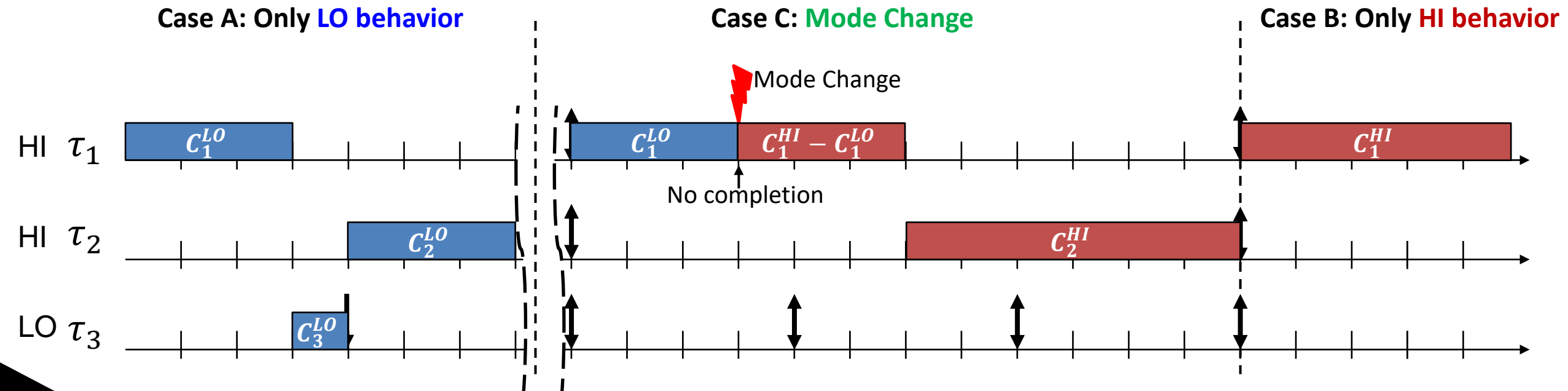


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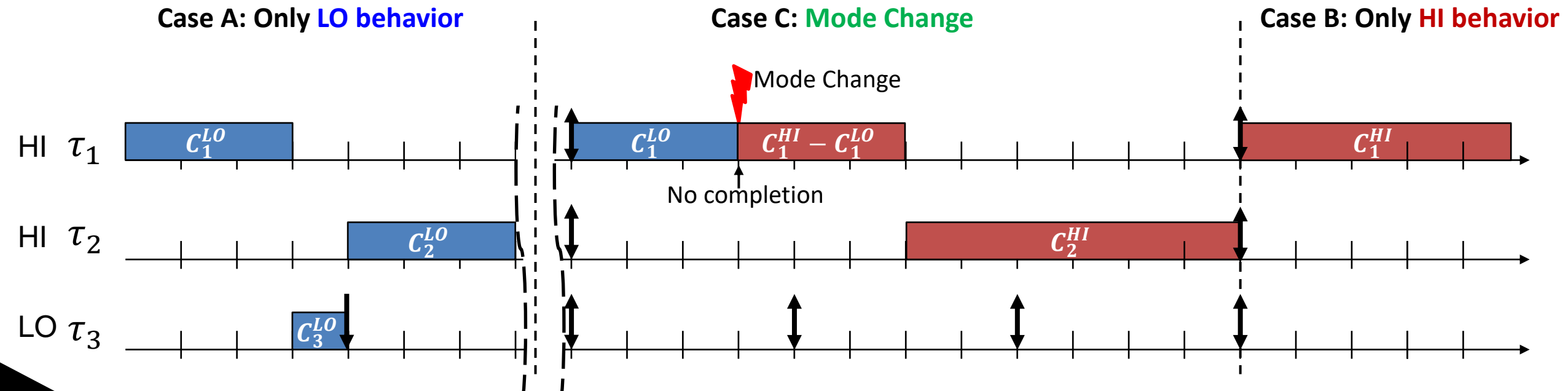
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C1. The **demand varies** depending on the system behavior

C2. It is **impossible to know** beforehand **when the mode change occurs**

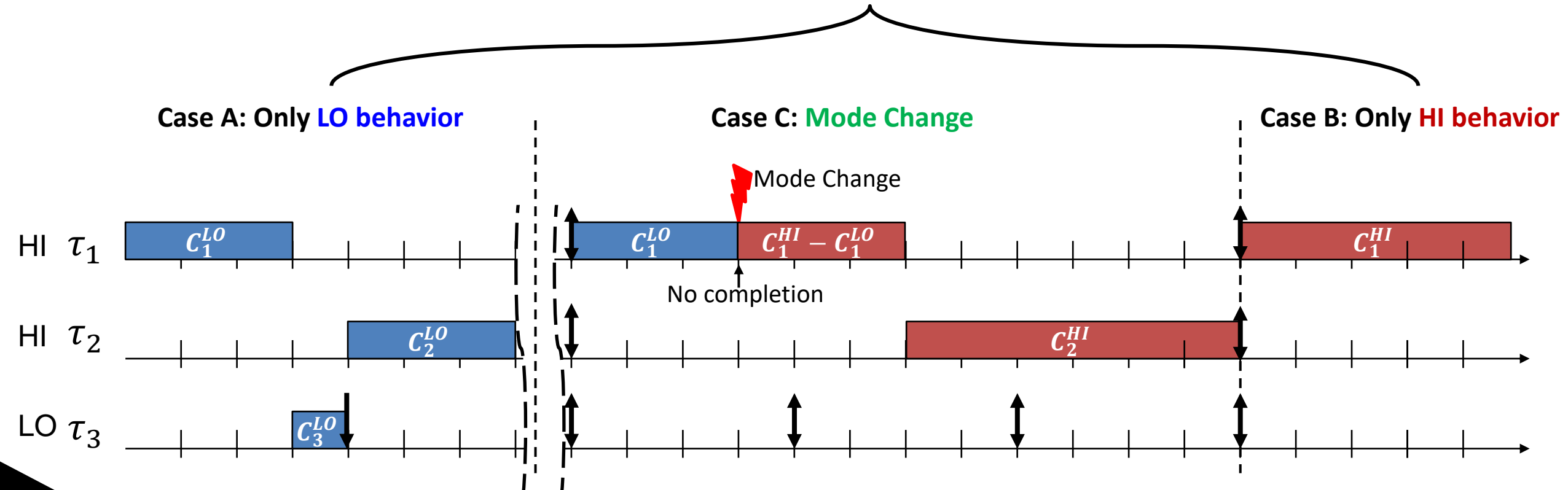


Necessary Feasibility of MC Task Systems



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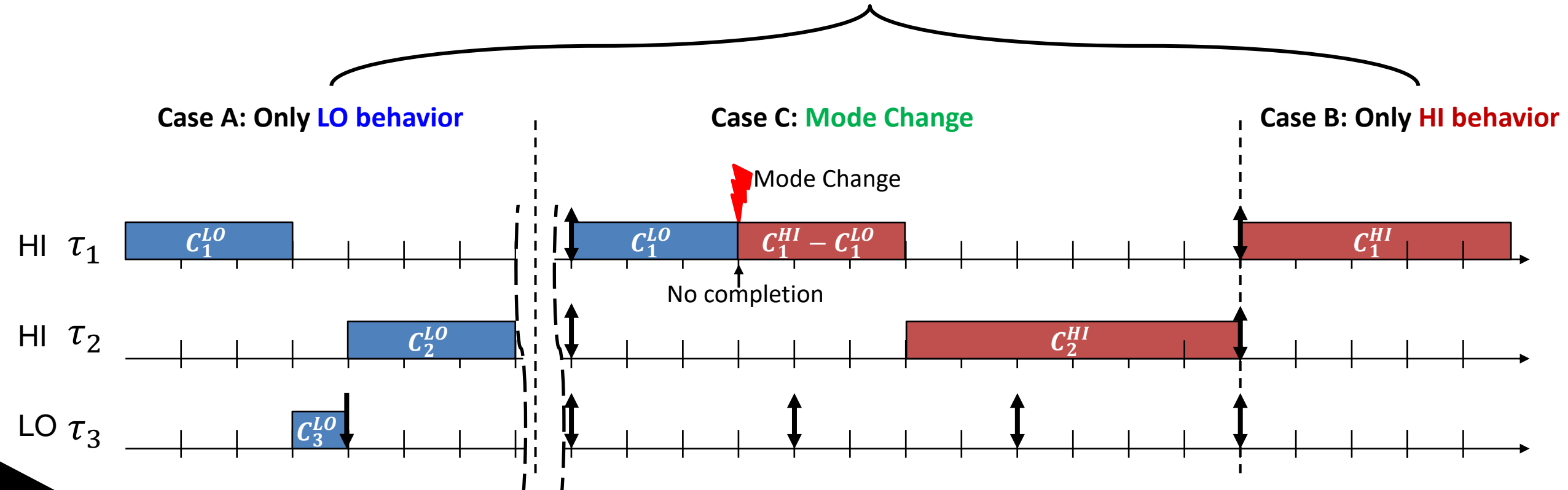
each scenario of Cases A & B \equiv a scenario of a single-criticality task system



Necessary Feasibility of MC Task Systems

the demand > the supply \Rightarrow infeasible

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Necessary Feasibility of MC Task Systems

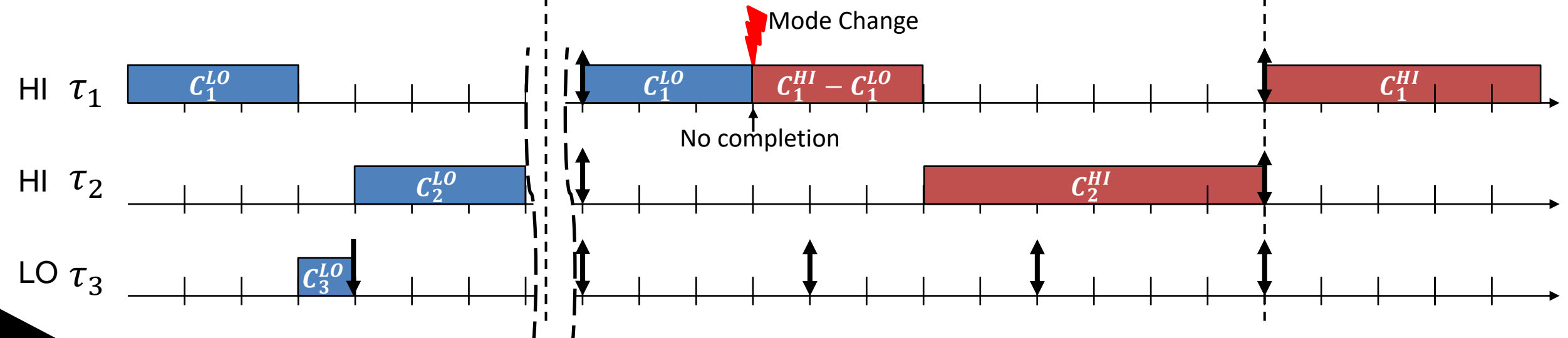
?



Case A: Only LO behavior

Case C: Mode Change

Case B: Only HI behavior



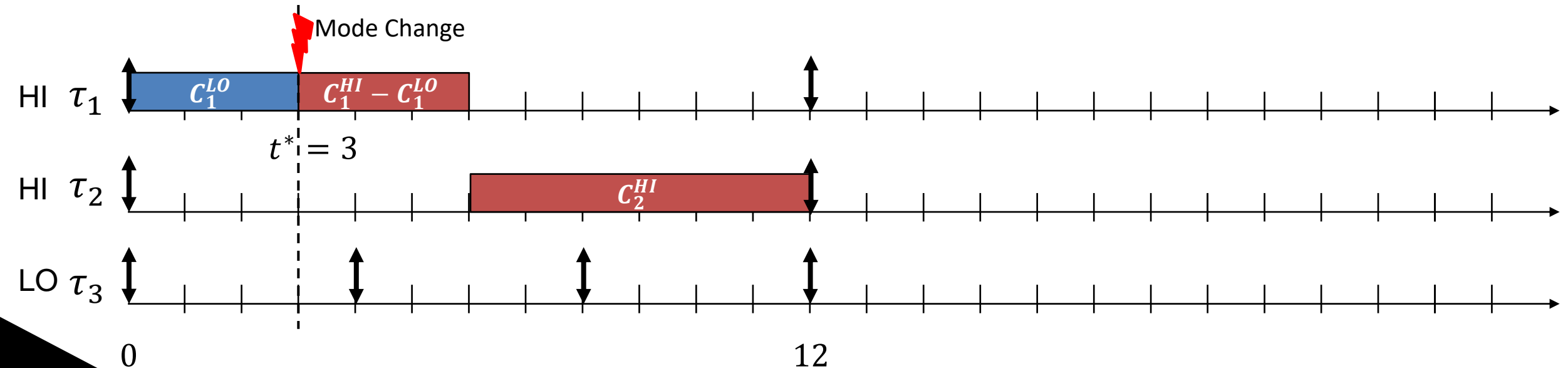
Necessary Feasibility of MC Task Systems

When the mode change occurs at $t^* = 3$

- The demand of τ_1 in $[0,12] = C_1^{HI}$
- The demand of τ_2 in $[0,12] = C_2^{HI}$
- The demand of τ_3 in $[0,12] = 0$

The demand in $[0,12] = 12 \leq$ the supply in $[0,12]$

Case C: **Mode Change**



Necessary Feasibility of MC Task Systems

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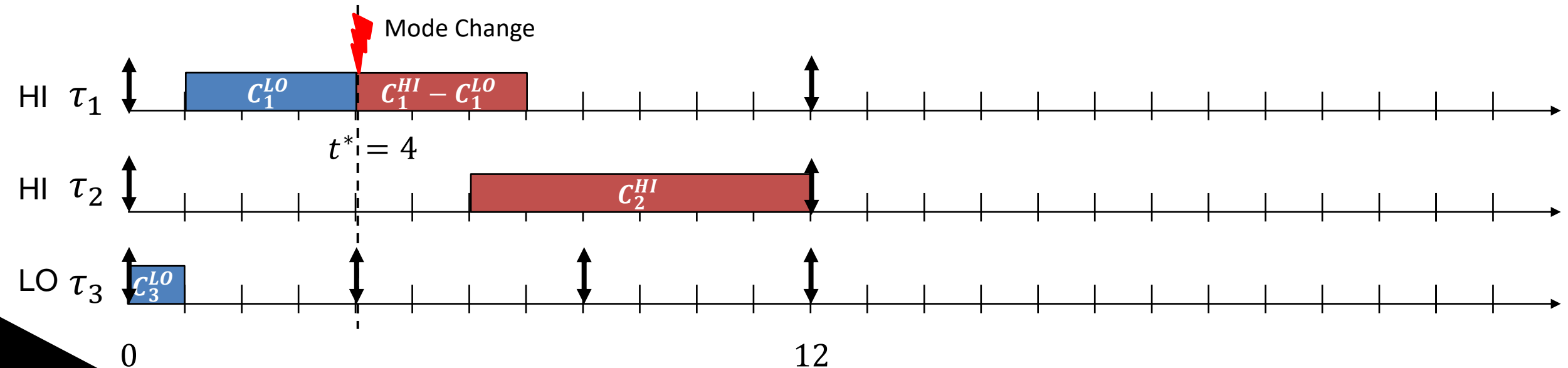
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When the mode change occurs at $t^* = 4$

- The demand of τ_1 in $[0,12] = C_1^{HI}$
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- The demand of τ_3 in $[0,12] = 1$

The demand in $[0,12] = 13 >$ the supply in $[0,12]$

Case C: Mode Change



Key Observations and Challenges

Key observations

- O1. The contribution of each LO job to **the demand varies** with the mode change instant
- O2. It is **impossible to calculate the demand** without specifying the mode change instant
- O3. The demand $>$ the supply in a case **does not necessarily yield infeasibility** of the scenario

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Challenges

- Q1. How to **characterize and calculate the demand** in an interval that changes depending on the mode change instant? (from O1 & O2)
- Q2. What is **the meaning of the demand $>$ the supply** in an interval when the mode change instant is given? (from O3)
- Q3. How to **derive a necessary feasibility condition** without assuming the mode change instant is given? (from O2 & O3)

Our Approach

Q1. How to characterize and calculate the demand ?

Q2. What is the meaning of the demand $>$ the supply ?

Q3. How to derive a necessary feasibility condition ?

Our Approach

Q1. How to characterize and calculate the demand ?

- Specify **a range of mode change instant** without the target scheduling algorithm (Lemma 4)
- **Select two sub-intervals** based on a mode change instant t^*
- **Calculate the demand** in the target sub-intervals (Lemmas 5,6,7)

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Q2. What is **the meaning of the demand > the supply** ?

- Compare the total demand with the total supply in the target sub-intervals
- **Judge the infeasibility of the mode change instant t^*** (Lemma 8)

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Q3. How to **derive a necessary feasibility condition** ?

- Repeat Lemma 8 for all t^* in the mode change instant range
- **Check there exists no feasible mode change instant** (Infeasibility of the task set) (Theorem 1)

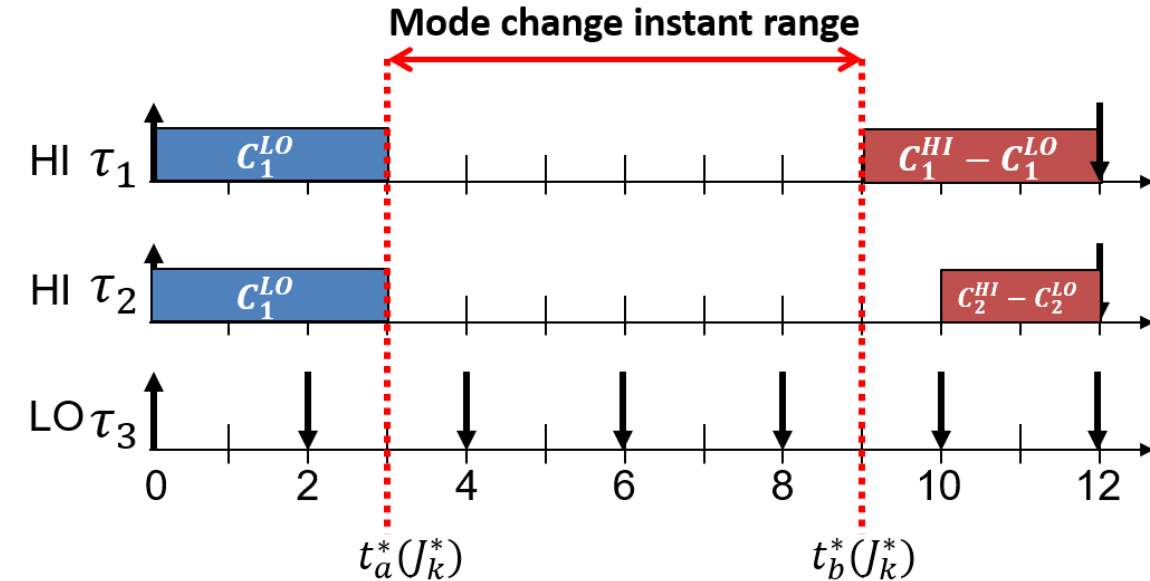
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(the job with the earliest release time among all HI jobs whose execution requirement is strictly larger than LO WCET)

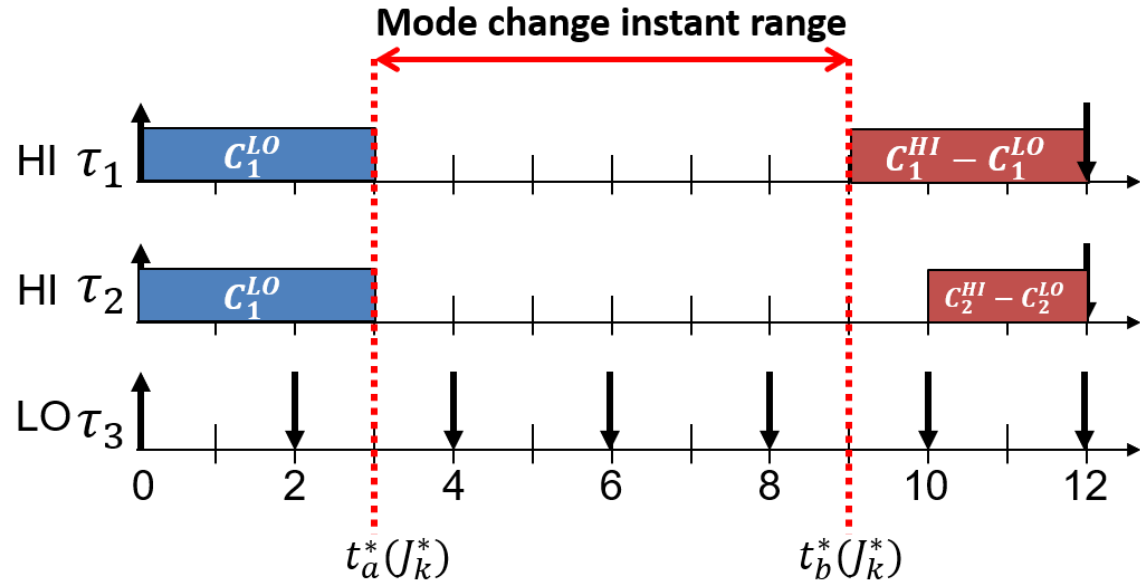
(b) Specify mode change instant range in $t^* \in [3, 9]$

(Lemma 4)

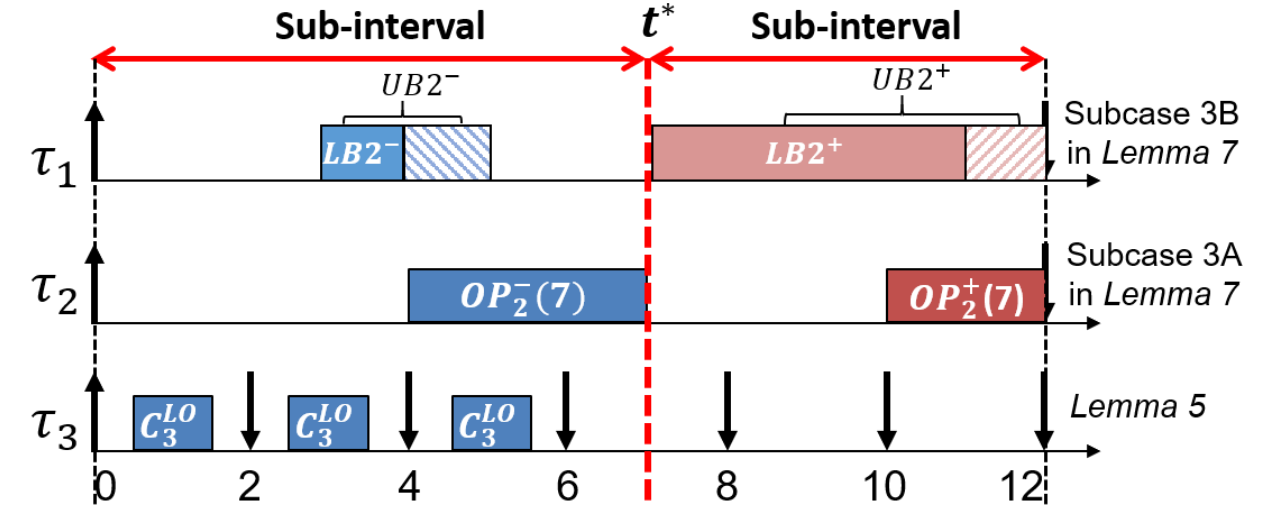


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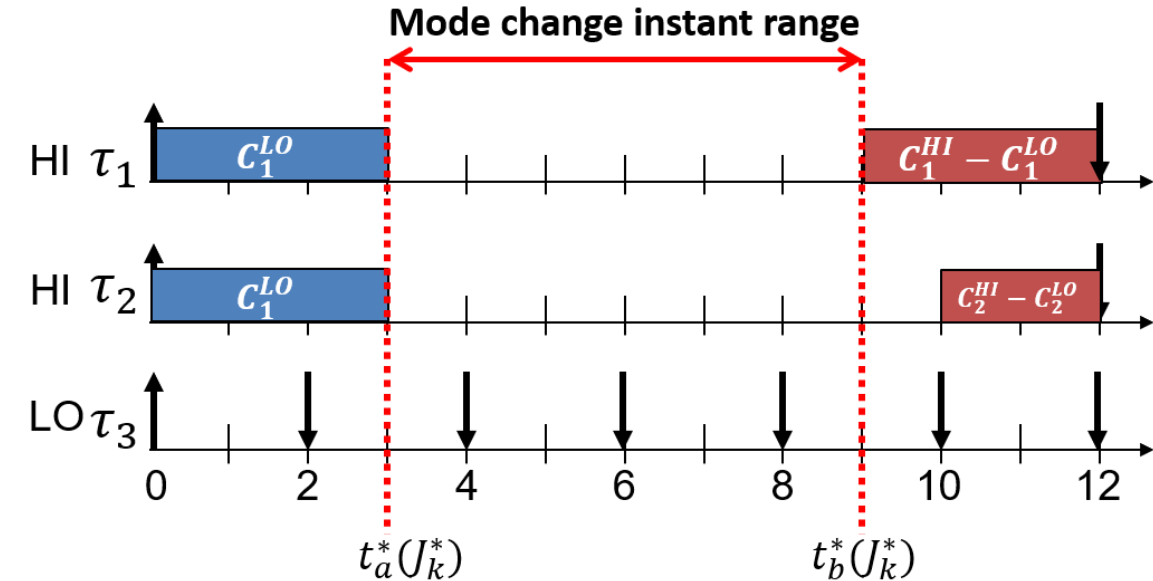


- (c) Given t^* , select sub-intervals $[0, 7]$ and $[7, 12]$**

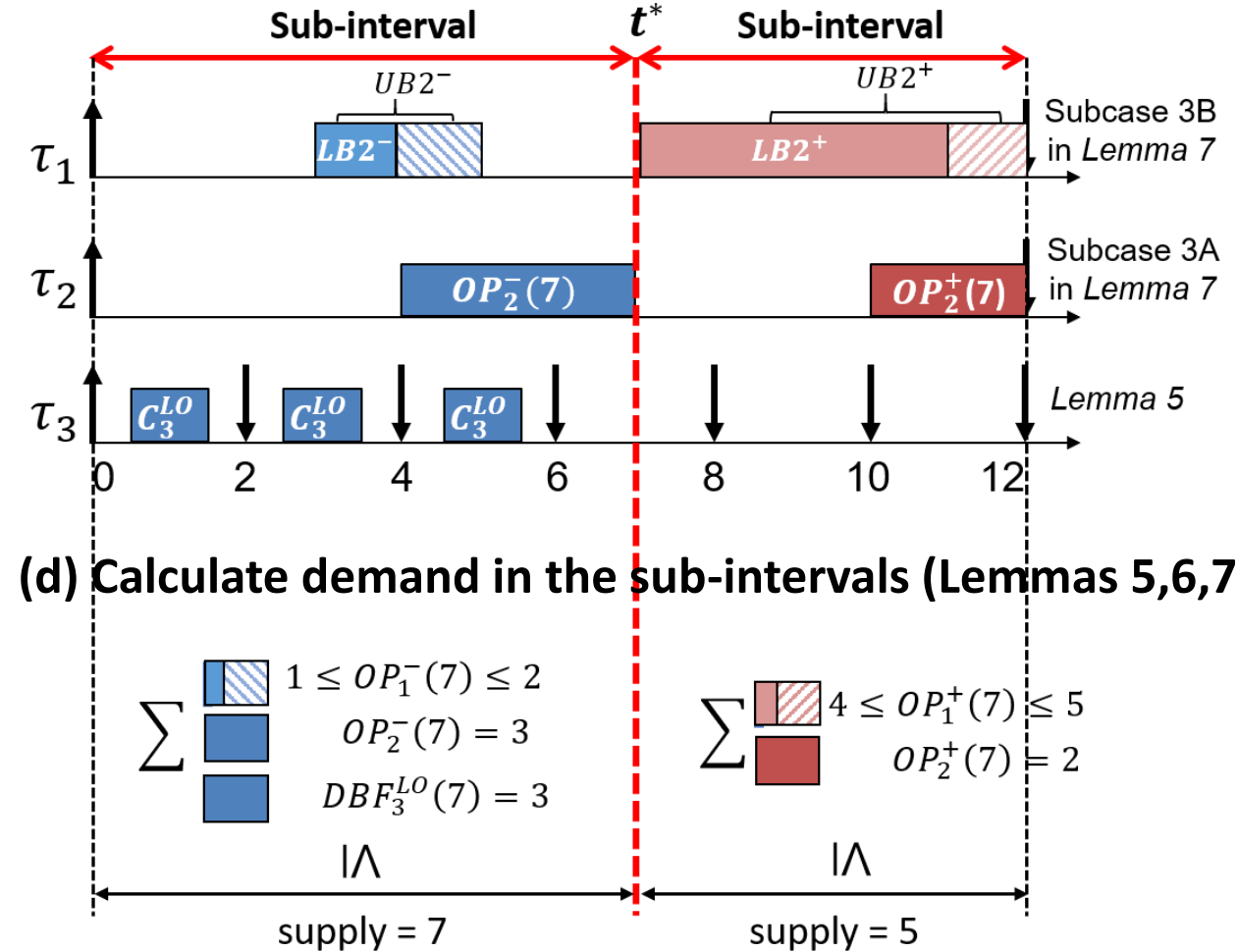


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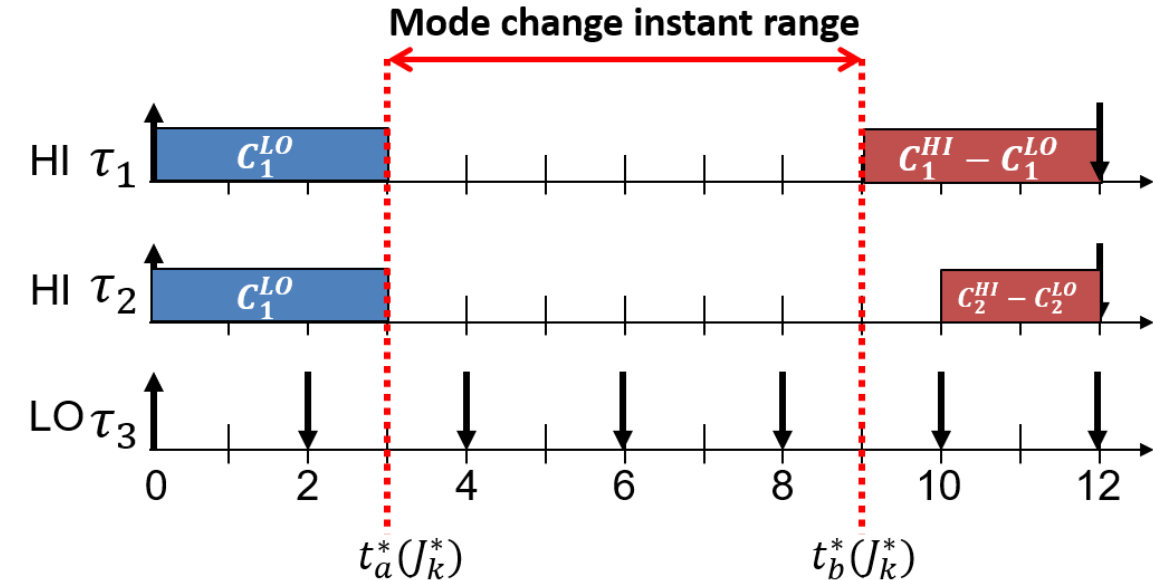
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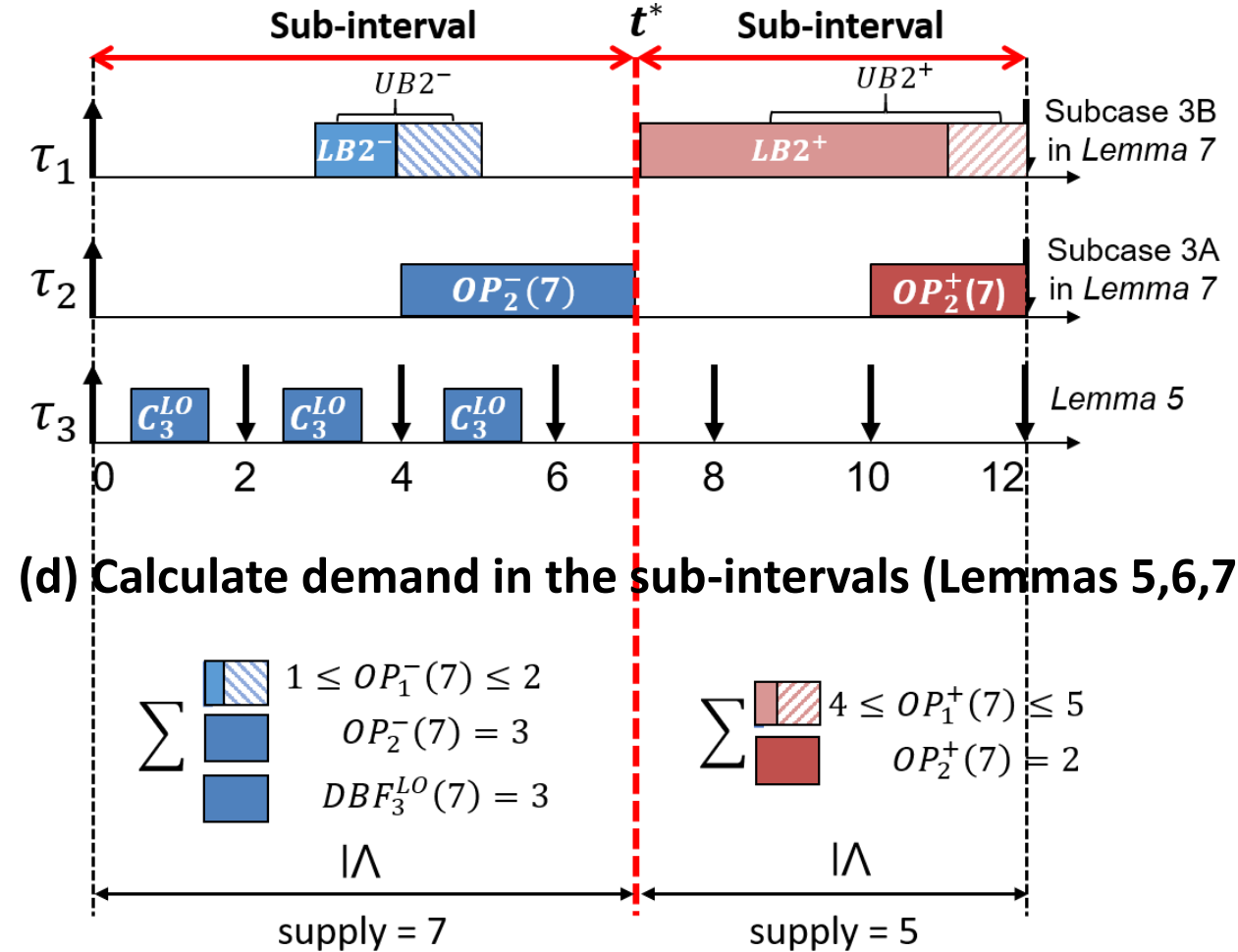
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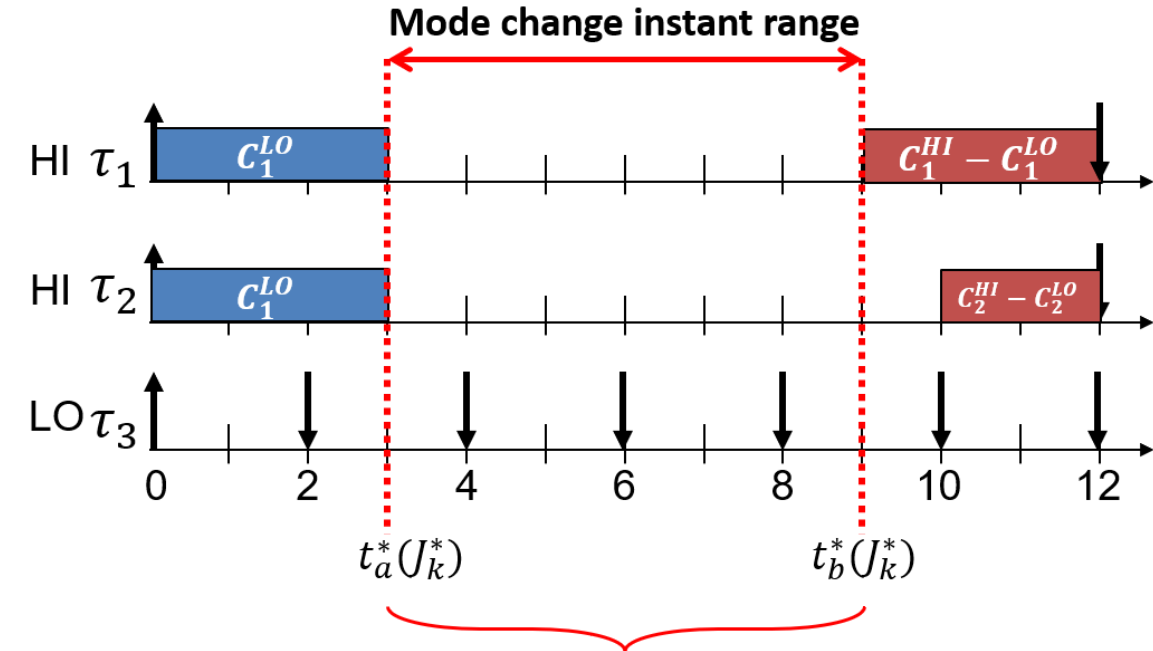


- (d) Calculate demand in the sub-intervals (Lemmas 5,6,7)

- (e) Check infeasibility of the mode change instant t^*
(Lemma 8)

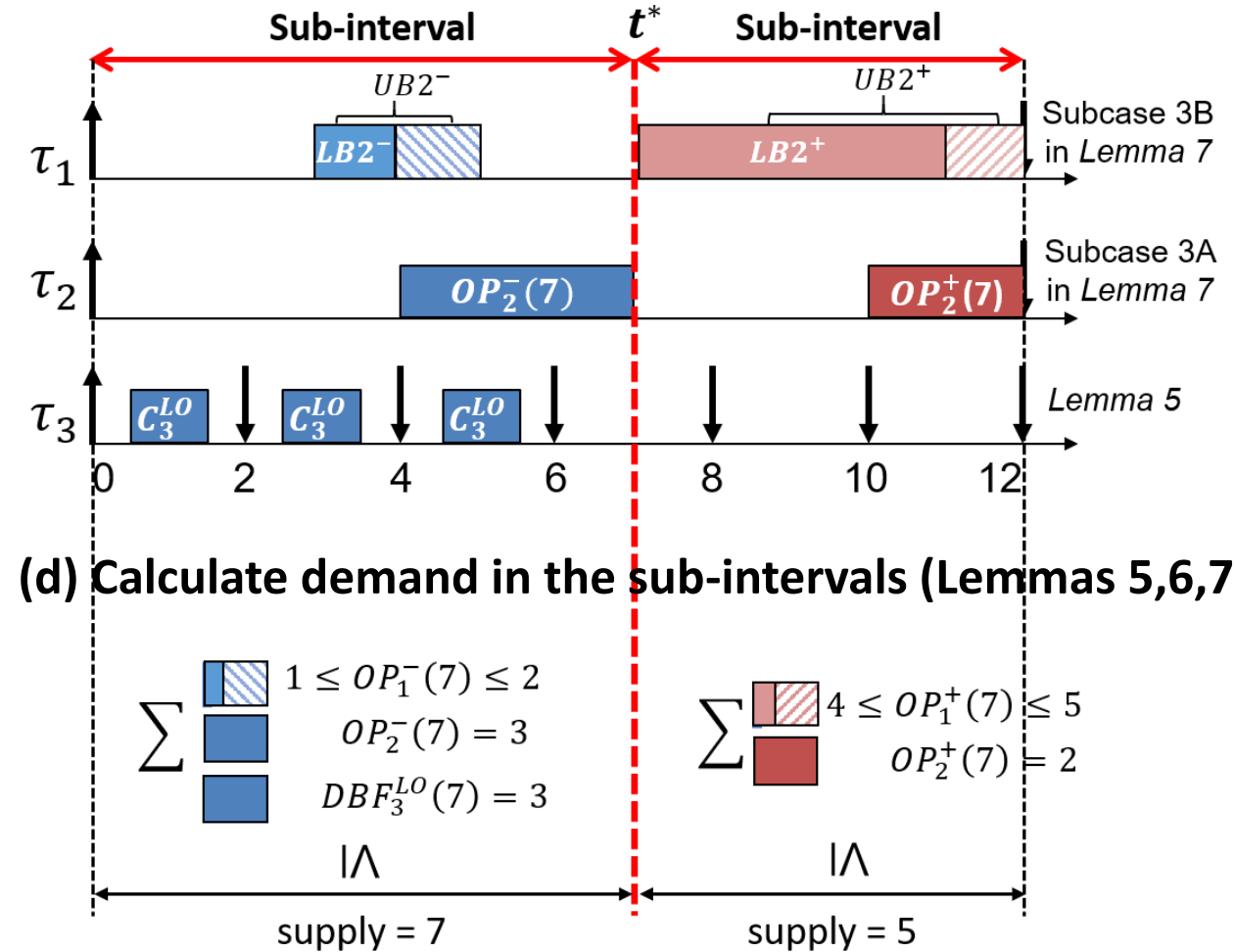
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- (b) Specify mode change instant range in $t^* \in [3,9]$ (Lemma 4)



- (f) Repeat for all $t^* \in [3,9]$, check no feasible mode change instant implying the task set's infeasibility (Theorem 1)

- (c) Given t^* , select sub-intervals $[0,7]$ and $[7,12]$

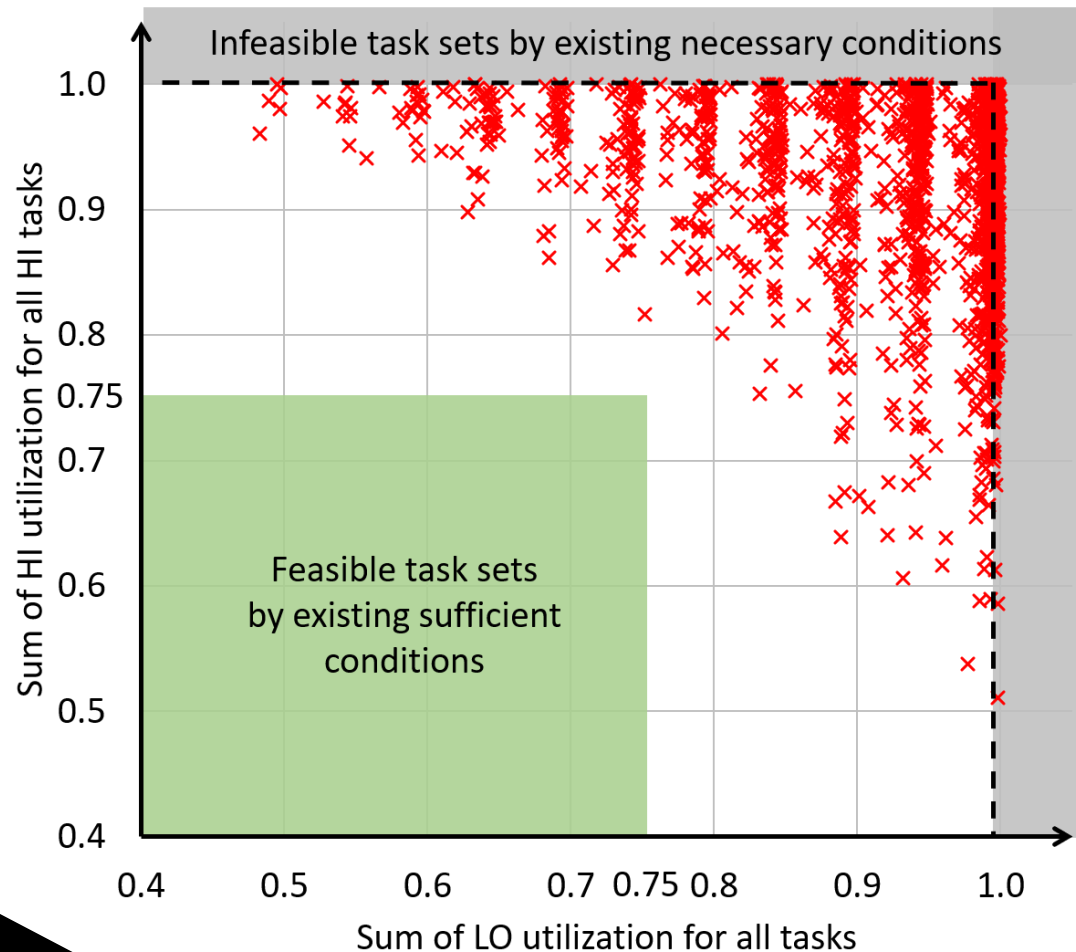


- (d) Calculate demand in the sub-intervals (Lemmas 5,6,7)

- (e) Check infeasibility of the mode change instant t^* (Lemma 8)

Evaluation

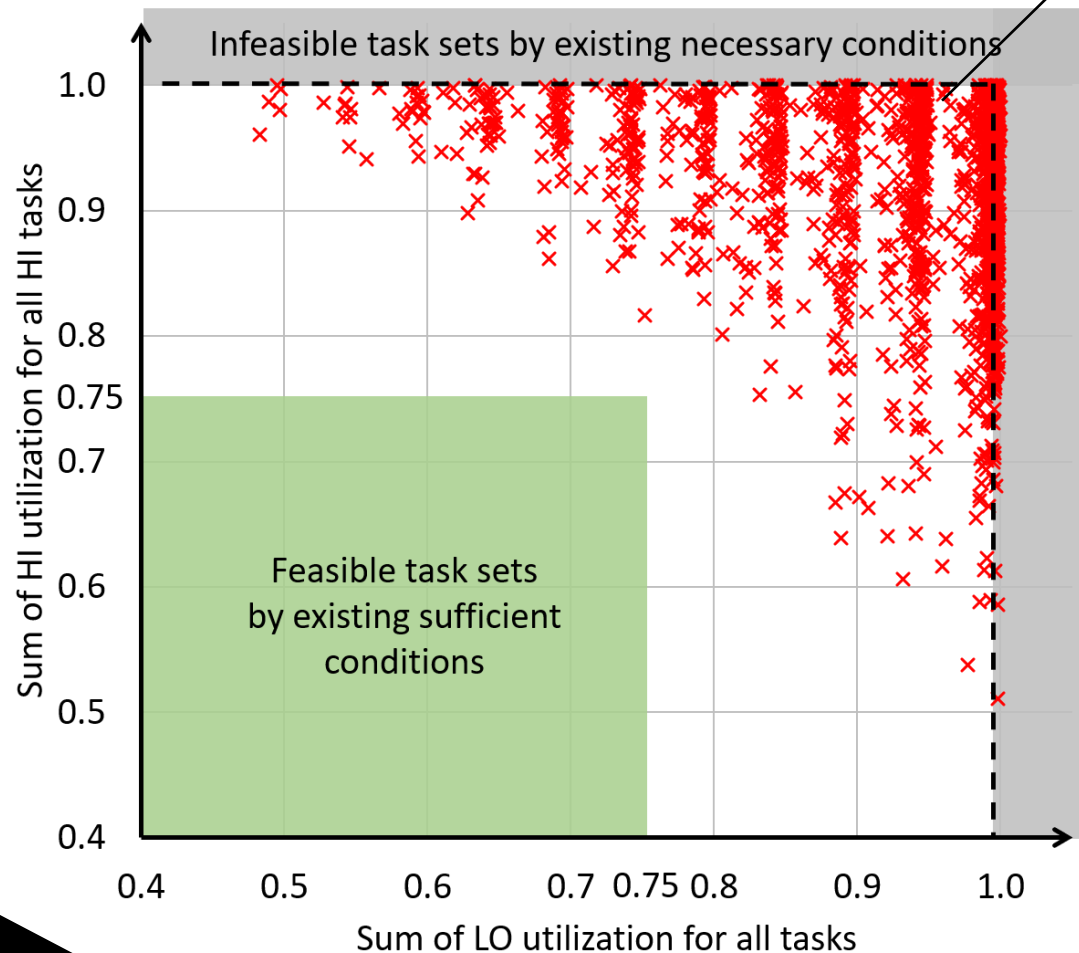
- Implicit-deadline task sets



Evaluation

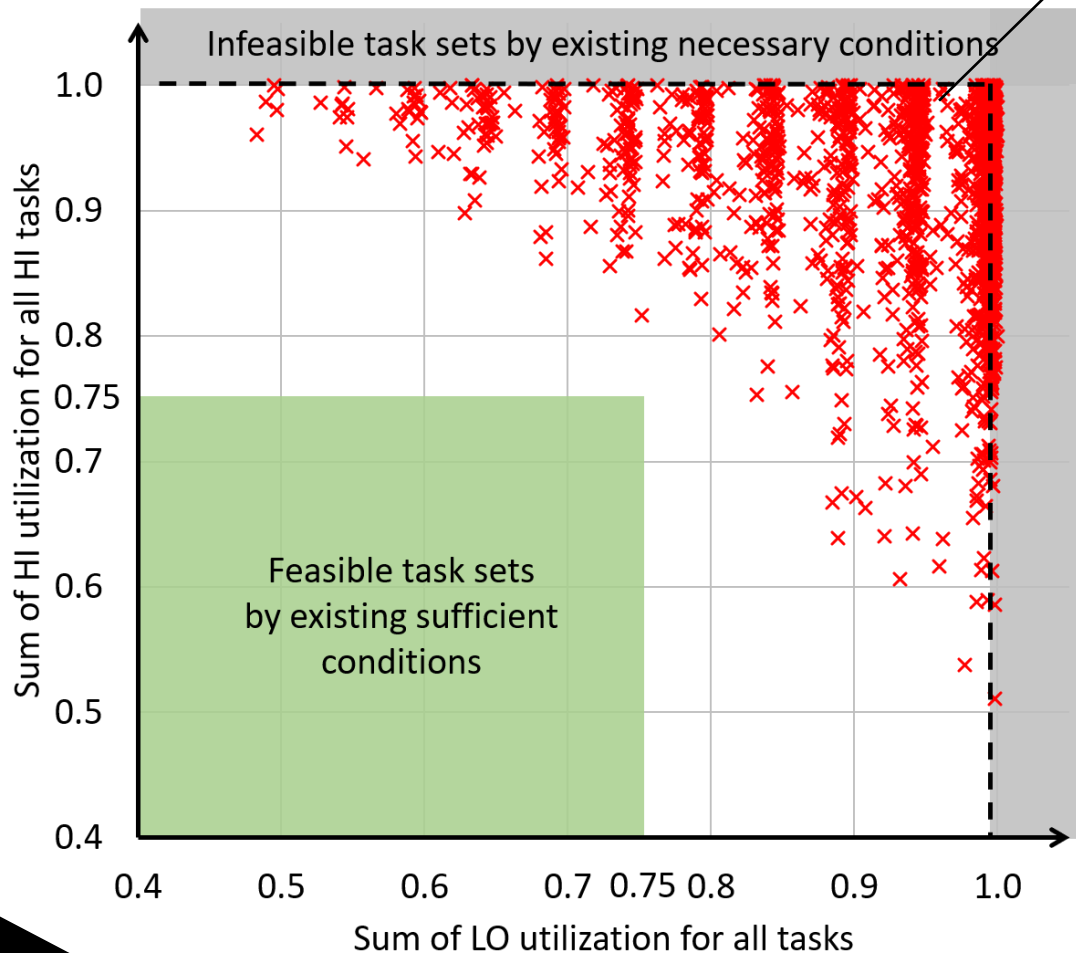
- Implicit-deadline task sets

✕ Task sets proven infeasible by this paper



Evaluation

■ Implicit-deadline task sets

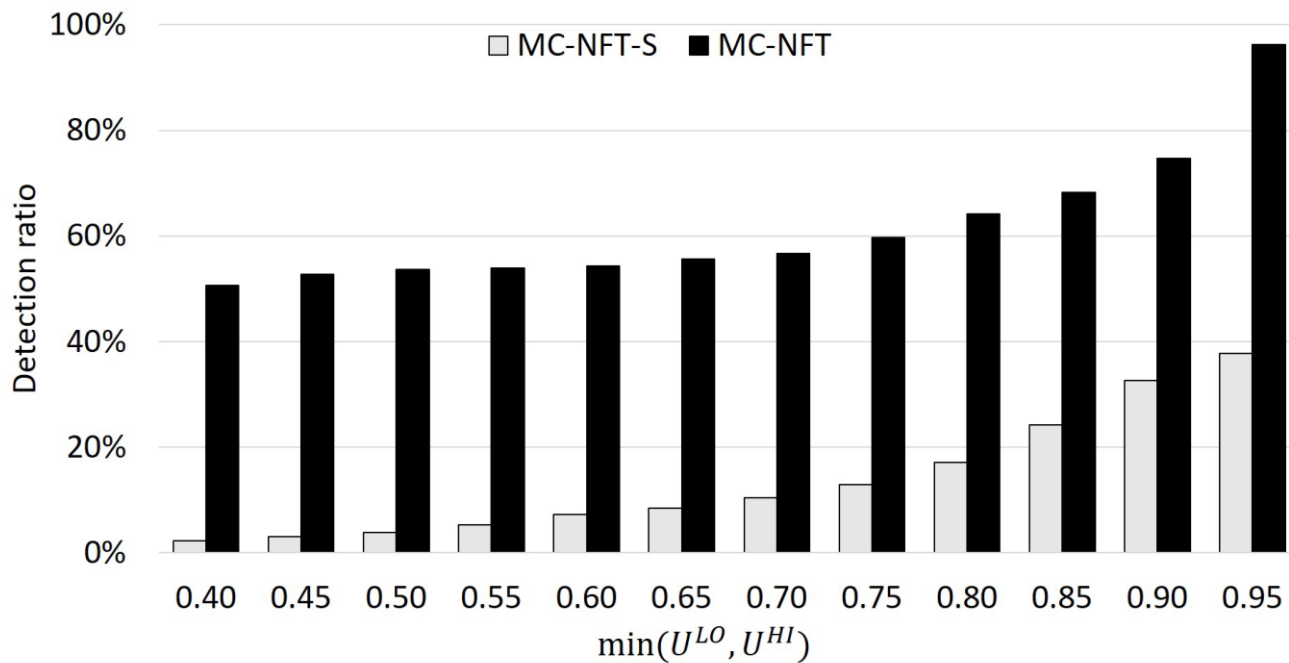


✕ Task sets proven infeasible by this paper

- Find several additional infeasible task sets **over a wider range** of LO and HI total utilization **which have been proven neither feasible nor infeasible by any existing studies**
- Identify significantly more infeasible task sets as LO and HI total utilization become close to 1.0

Evaluation

Constrained-deadline task sets



- Exhibit high capability in finding infeasible task sets

MC-NFT: 56% task sets proven infeasible

MC-NFT-S: 8.2% task sets proven infeasible

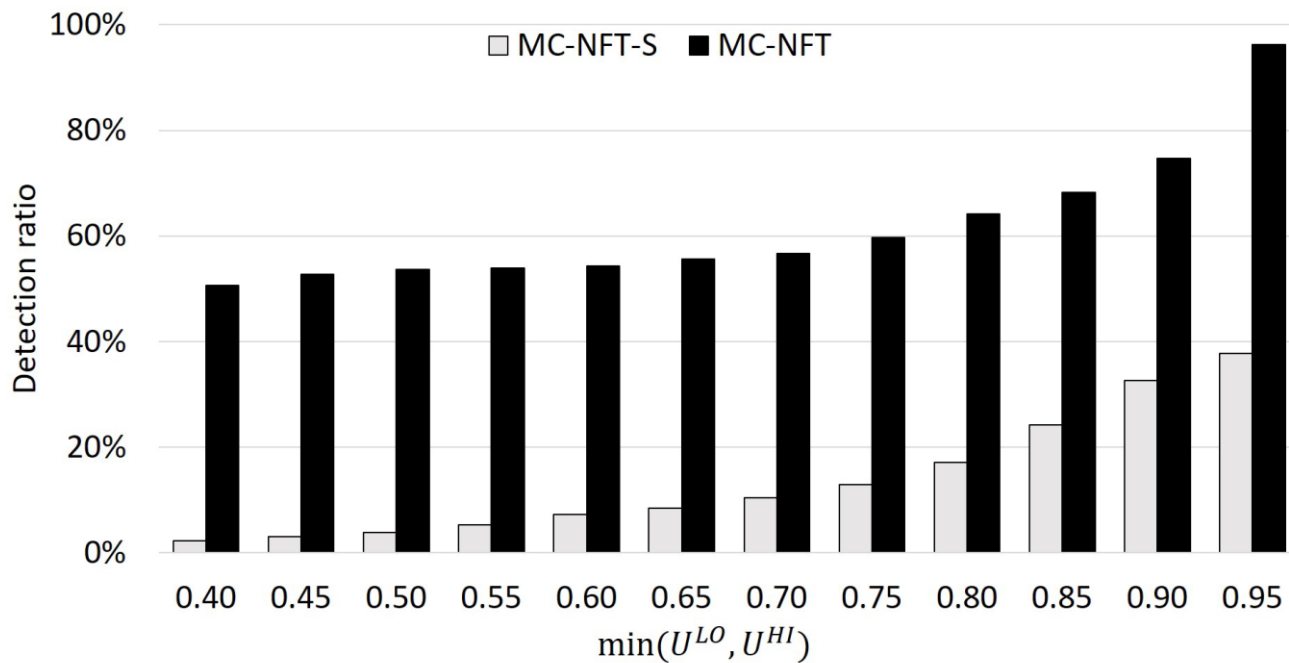
among task sets which have been proven
neither feasible nor infeasible
by any existing studies

MC-NFT: collective necessary feasibility test in Theorem 2

MC-NFT-S: simplified version of MC-NFT in Theorem 3

Evaluation

Constrained-deadline task sets



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MC-NFT: 56% task sets proven infeasible

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**Benefit of dealing with unique issues
in MC task systems**

MC-NFT: collective necessary feasibility test in Theorem 2

MC-NFT-S: simplified version of MC-NFT in Theorem 3

Conclusion

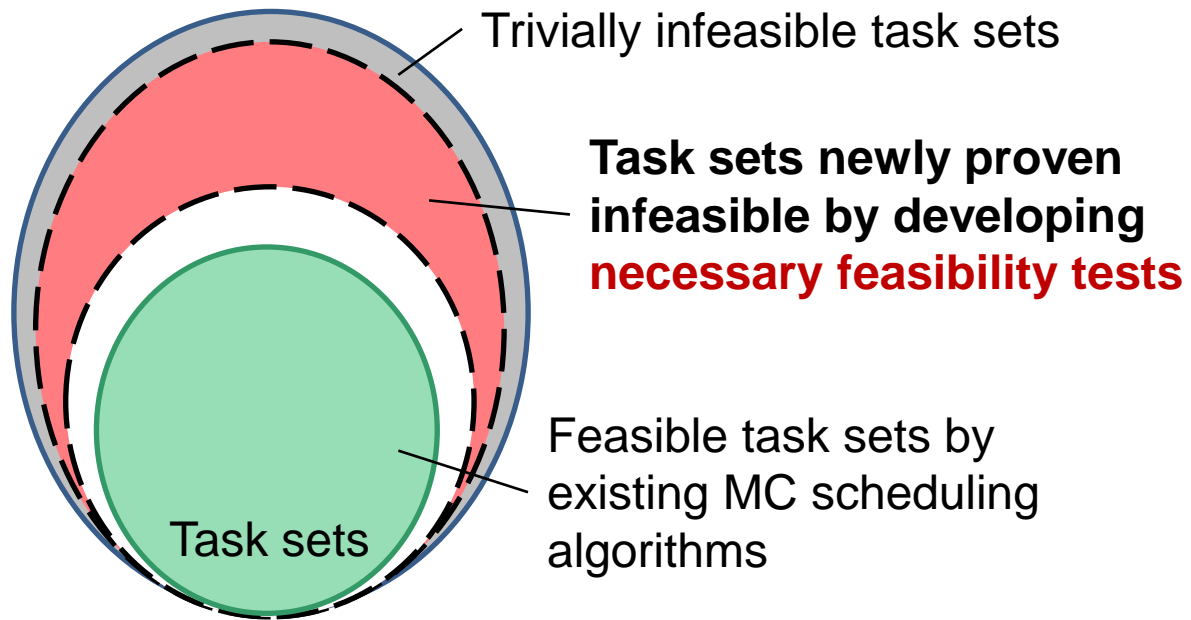
The paper provides

tight necessary feasibility tests for mixed-criticality (MC) systems on uniprocessor

Conclusion

The paper provides

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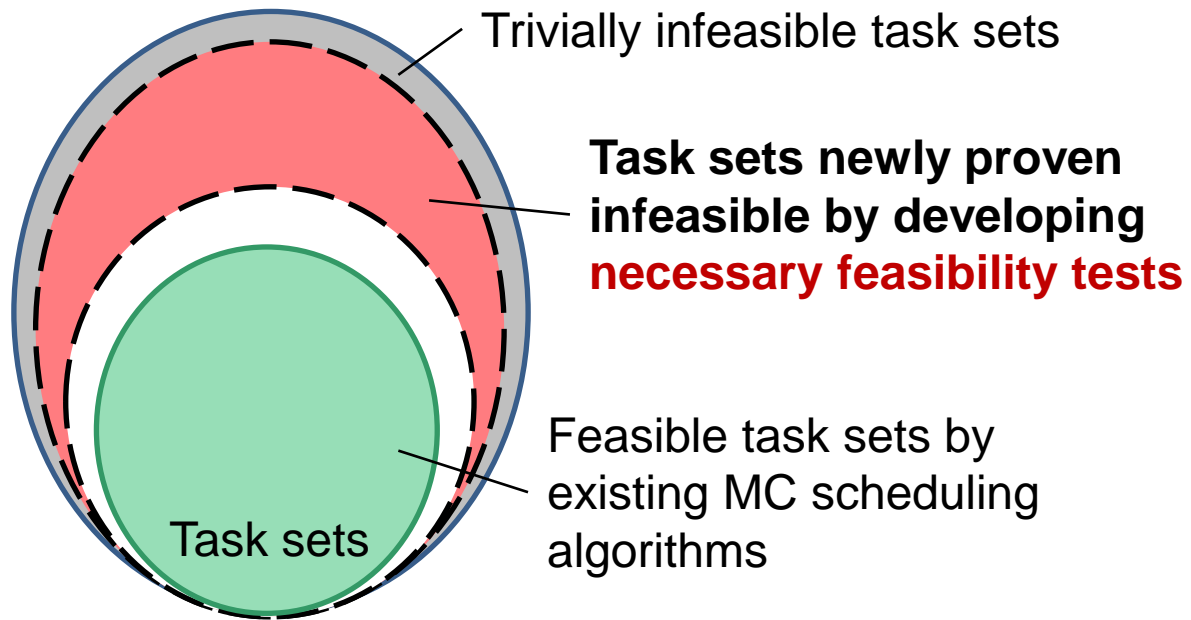


The first study that yields non-trivial results for **MC necessary feasibility**

Conclusion

The paper provides

tight necessary feasibility tests for mixed-criticality (MC) systems on uniprocessor



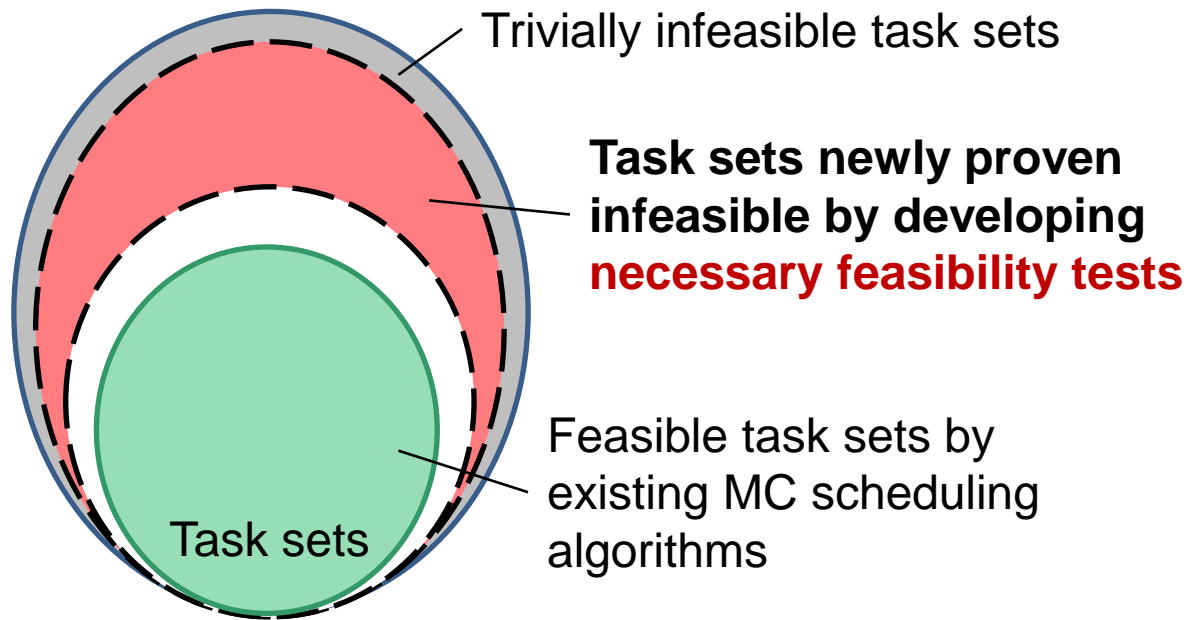
The first study that yields non-trivial results for **MC necessary feasibility**

- ➡ Investigate **characteristics of MC systems** in terms of necessary feasibility
- ➡ Identify **new challenges** posed by such characteristics of MC task systems
- ➡ Establish **foundations** of necessary feasibility tests for MC task systems

Conclusion

The paper provides

tight necessary feasibility tests for mixed-criticality (MC) systems on uniprocessor



The first study that yields non-trivial results for **MC necessary feasibility**

- ➡ Investigate **characteristics of MC systems** in terms of necessary feasibility
- ➡ Identify **new challenges** posed by such characteristics of MC task systems
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Reduce a gap between the task sets proven feasible and that proven infeasible